Homework

1. The diffusion problem for the density $\rho(x,t)$ in a 1D box is defined by

$$\frac{\partial}{\partial t}\rho(x,t) = D\frac{\partial^2}{\partial x^2}\rho(x,t) \tag{1}$$

with the boundary conditions

$$\frac{\partial \rho}{\partial x}|_{x=\pm s/2} = 0. \tag{2}$$

(a) Show that the limiting density $lim_{t\longrightarrow\infty}\rho(x,t)$ is uniform.
(b) Calculate the relaxation rate to this density.

2. Calculate the Ruelle-Pollicott resonances for the baker map (see H.H. Hasegawa and W.C. Saphir, Phys. Rev. A **46**, 7401 (1992)).

3. find the fixed points of the standard map and classify them by their stability.

4. Calculate the inverse localization length $\gamma(E) = 1/\xi$ for the Lloyd model

$$\epsilon_i u_i + u_{i+1} + u_{i-1} = E u_i \tag{3}$$

where the distribution of diagonal energies is a Lorentzian (Cauchy), namely

$$P(\epsilon_i) = \frac{1}{\pi}\frac{\delta}{\epsilon_i^2 + \delta^2}. \tag{4}$$

(see K. Ishii, Prog. Theor. Phys. Suppl. **53**, 77 (1973)).

Answer: $\cosh\gamma(E) = \frac{1}{4}\left[\sqrt{(2+E)^2+\delta^2} + \sqrt{(2-E)^2+\delta^2}\right]$.

5. Prove the Thouless formula

$$\gamma(E) = \int \rho(x)\ln|E-x|\,dx \tag{5}$$

where $\rho(x)$ is the density of states (per site) for the model

$$\epsilon_i u_i + u_{i+1} + u_{i-1} = E u_i$$

(see D.J. Thouless, J. Phys. C **5**, 77 (1972)).

6. Calculate the conductance of the one dimensional ideal wire at zero temperature.

7. Assume that φ is a random variable, uniformly distributed in the interval $[0,\ 2\pi]$. What is the distribution of $y = \tan\varphi$?

8. Choose $u^{\pm}(\theta) = \mathrm{e}^{\mp i \frac{V(\theta)}{2}} \bar{u}(\theta)$, where $V(\theta) = k\cos\theta$, and show that for a state with a quasienergy ω

$$\sum_r J_{n-r}\left(\frac{k}{2}\right) \sin\left(\frac{1}{2}\left[(\omega - \frac{\tau}{2}n^2) - \pi(n-r)\right]\right) \bar{u}_r = 0 \tag{6}$$

(see D.L. Shepelyansky, Phys. Rev. Lett. **56**, 677 (1986); Physica D **28** 103 (1987), notations will be defined during the lecture).

9. (a) Find the quasienergies and quasienergy states for the model

$$i\frac{\partial\psi}{\partial t} = \tau\left(-i\frac{\partial}{\partial\theta}\right)\psi + V(\theta)\sum_m \delta(t-m). \tag{7}$$

(see D.R. Grempel, S. Fishman and R.E. Prange, Phys. Rev. Lett. **49**, 833 (1982)).

(b) Find the specific solutions for $V(\theta) = k\cos\theta$.
(c) Find the specific solutions for $V(\theta) = -2\arctan\left[k\cos\theta - E\right]$.

10. Use 9(c) to find the eigenstates for the Maryland model

$$\tan\left[(\omega - m\tau)/2\right] u_m + k\left(u_{m+1} + u_{m-1}\right) = E u_m\ . \tag{8}$$

11. Assume that in the model $\epsilon_n u_n + u_{n+1} + u_{n-1} = E u_n$, the ϵ_n can be considered small. Show that in the second order in the $\{\epsilon_n\}$ the inverse localization length is

$$\gamma(E) = lim_{N\longrightarrow\infty}\frac{1}{8\sin^2 k}\frac{1}{N}\left|\sum_{n=1}^{N}\epsilon_n \sin k\right|, \tag{9}$$

where the eigenvalues of the unperturbed problem are $E = 2\cos k$. Use the Thouless formula (problem 5) and show first that Green's function of the unperturbed system G^0_{ij} is proportional to $\mathrm{e}^{i|i-j|k}$ (see D.J. Thouless, J. Phys. C **6**, L49 (1973)).