Dissipation and Universal Response in Complex Quantum Systems

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My lectures will consider two closely related aspects of complex quantum systems.

It is well known that the energy levels of complex quantum systems have 'universal' statistical properties: on small energy scales these are similar to those of random matrix ensembles. It is natural to ask whether the dynamics of complex quantum systems also has universal properties. The answer is affirmative: there is a 'universal' response to low frequency perturbations, characterised by a diffusion of occupation probability over adiabatic states.

It is also natural to ask how the universal dynamical response, namely 'energy diffusion', is related to experimentally observable dynamical processes. It turns out that energy diffusion is closely related to dissipation. I shall describe a theory of dissipation based upon energy diffusion. It is equivalent to conventional linear response theory in most cases, but I also describe a range of situations where the predictions of linear response theory fail dramatically.

Lecture 1: Energy diffusion. Spectral universality and random matrices. Dynamical universality, energy diffusion, and dimensionless parameters characterising dynamics. Introduction of parametric random matrix models as a laboratory for studying universal dynamic processes. Brief discussion of connections between energy diffusion and dissipation. Semiclassical estimates of matrix elements.

Lecture 2: Dissipation. Models for dissipative processes. Linear response theory. Detailed description of dissipation in condensed matter problems: master equation for transfer of energy from electromagnetic fields to electrons and then to phonons. A critique of the linear response approach. Energy diffusion theory as an alternative approach. Demonstration of equivalence of linear response and energy diffusion theories under typical circumstances. Brief discussion of 'semilinear response', a failure of linear response theory.

Lecture 3: Survey of dynamical processes. Discussion of the large number of 'universal' dynamical regimes which are possible in response to a low-frequency stimulus. These include the Gorkov-Eliashberg model, semilinear response, Landau-Zener response, adiabatic transport, 'dynamical localisation' and the 'fluctuation anomaly'. Discussion of a paradox relating to the fluctuation anomaly. Some new ideas on variable-range hopping, and its relation to stick-breaking processes.

Lecture 4: Two precise asymptotic theories. The semilinear response model and the Landau-Zener model will be considered in detail, deriving precise results for random matrix spectra. The latter requires developing some results in parametric random matrix theory, involving using the Kac-Rice procedure to count point singularities of parametric spectra.

Lecture 5: Semiclassical theories. Semiclassical methods can be used to estimate matrix elements and commutators which are required for quantum dynamical theories. Semiclassical ideas will be used to explain why the fluctuation anomaly is so hard to observe, despite being generic. I shall also relate semiclassical ideas to random matrix theories for spectra, by considering the dynamics of the energy levels under a diffusively varying perturbation of the Hamiltonian.