Time asymmetry in nonequilibrium fluctuations

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BELGIUM

Outline

1. Time asymmetry and entropy production

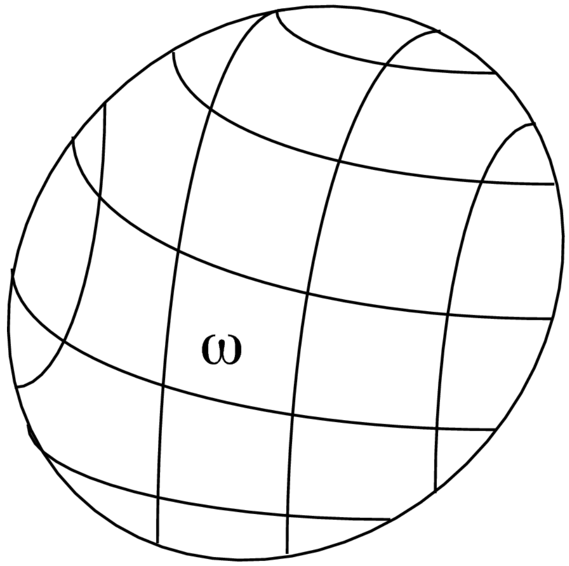
2. Experimental results

3. Information theory aspects

4. Summary

Dynamical evolution

phase space Γ



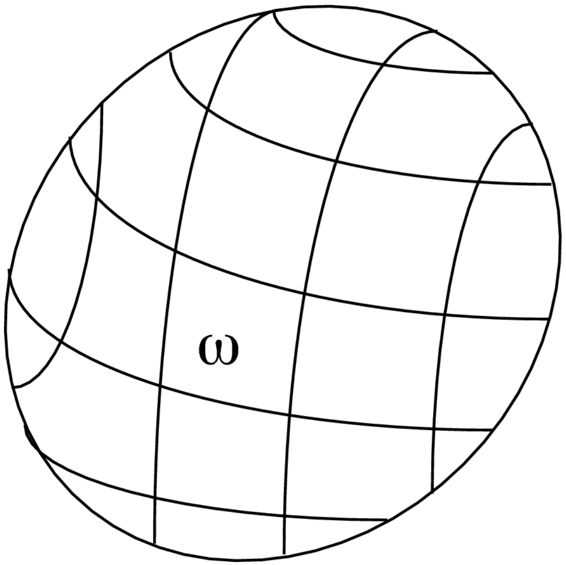
Flow in phase space Φ^t

Coarse-grained states ω

Observation at fixed time intervals τ

Dynamical evolution

phase space Γ



Observation at fixed time intervals τ

Coarse-grained states ω

Flow in phase space Φ^t

Trajectory $\omega_0 \omega_1 \cdots \omega_{n-1}$ occurs with probability

$$\mu(\omega_0 \omega_1 \cdots \omega_{n-1}) \equiv \mu(\Phi^{-(n-1)\tau} \omega_{n-1} \cap \cdots \cap \Phi^{-\tau} \omega_1 \cap \omega_0)$$

Breaking of time-reversal symmetry

Free motion of a particle

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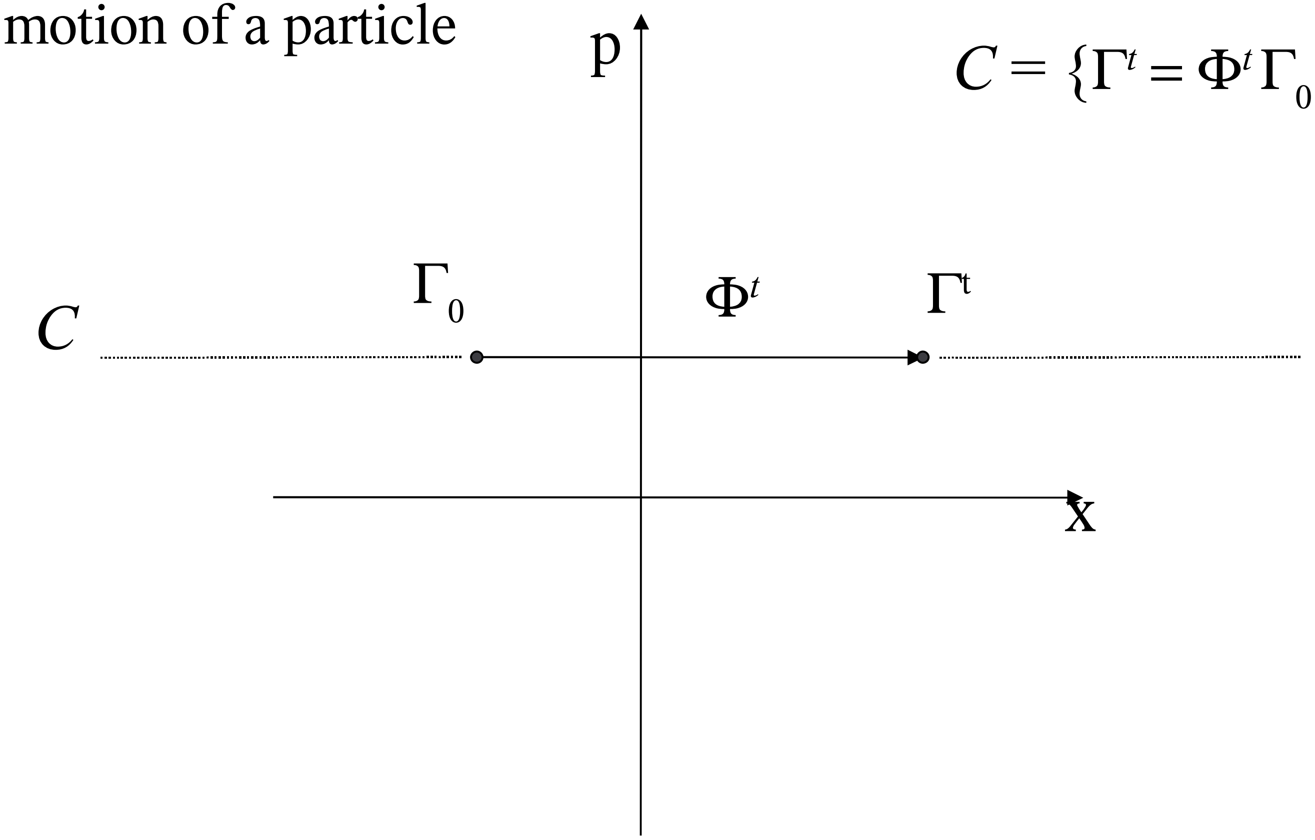
Free motion of a particle

$$C = \{\Gamma^t = \Phi^t \Gamma_0\}$$



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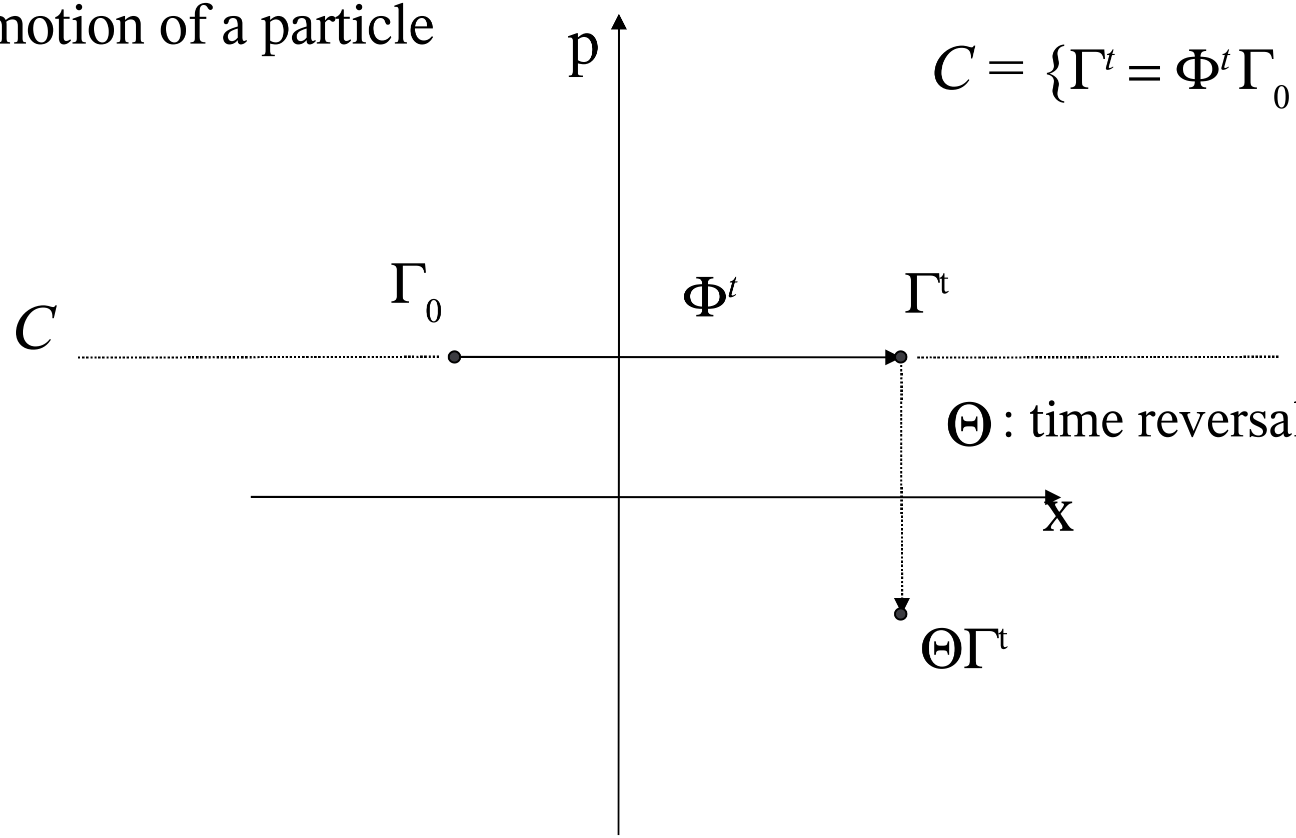

Θ : time reversal operator

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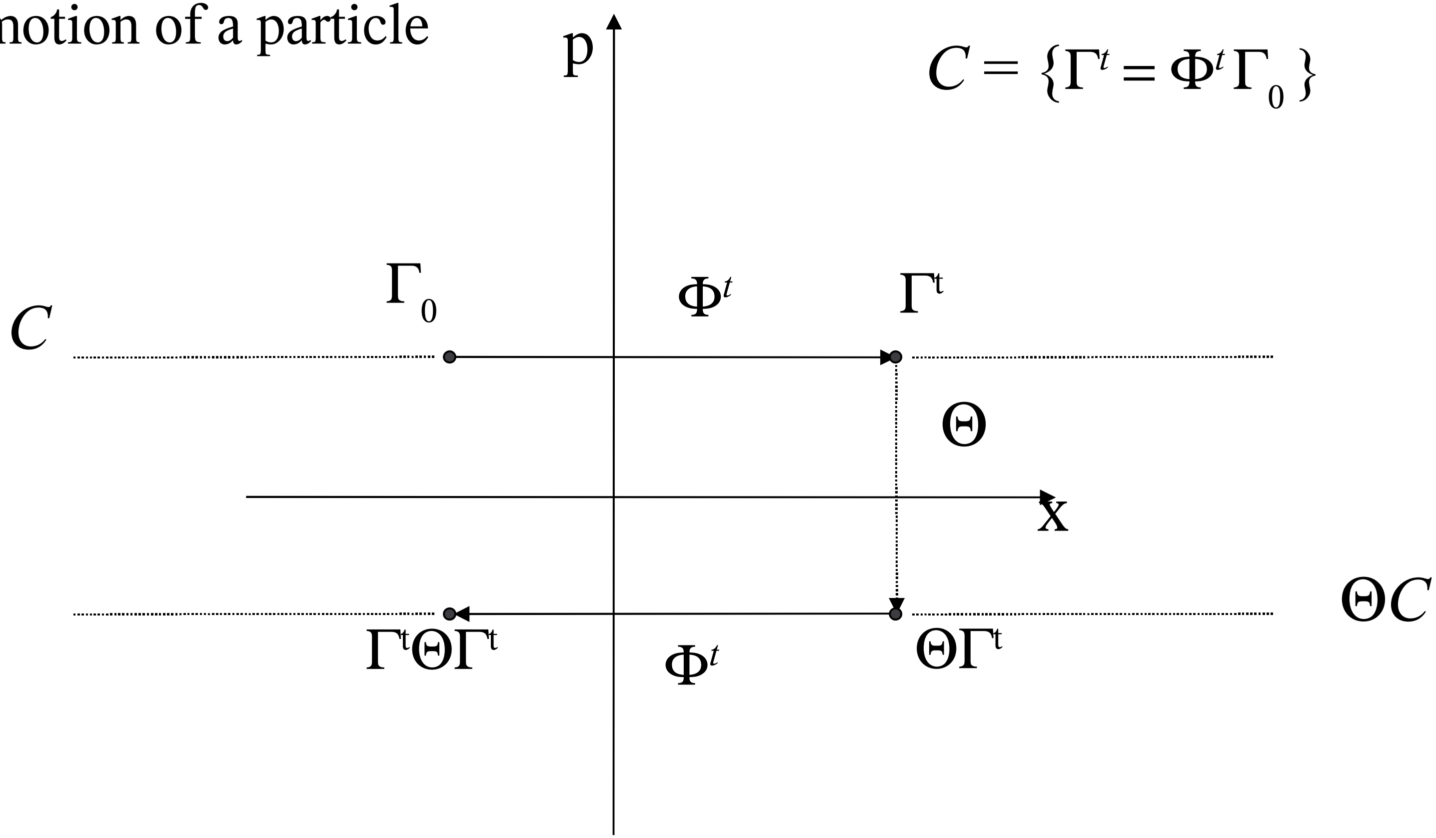

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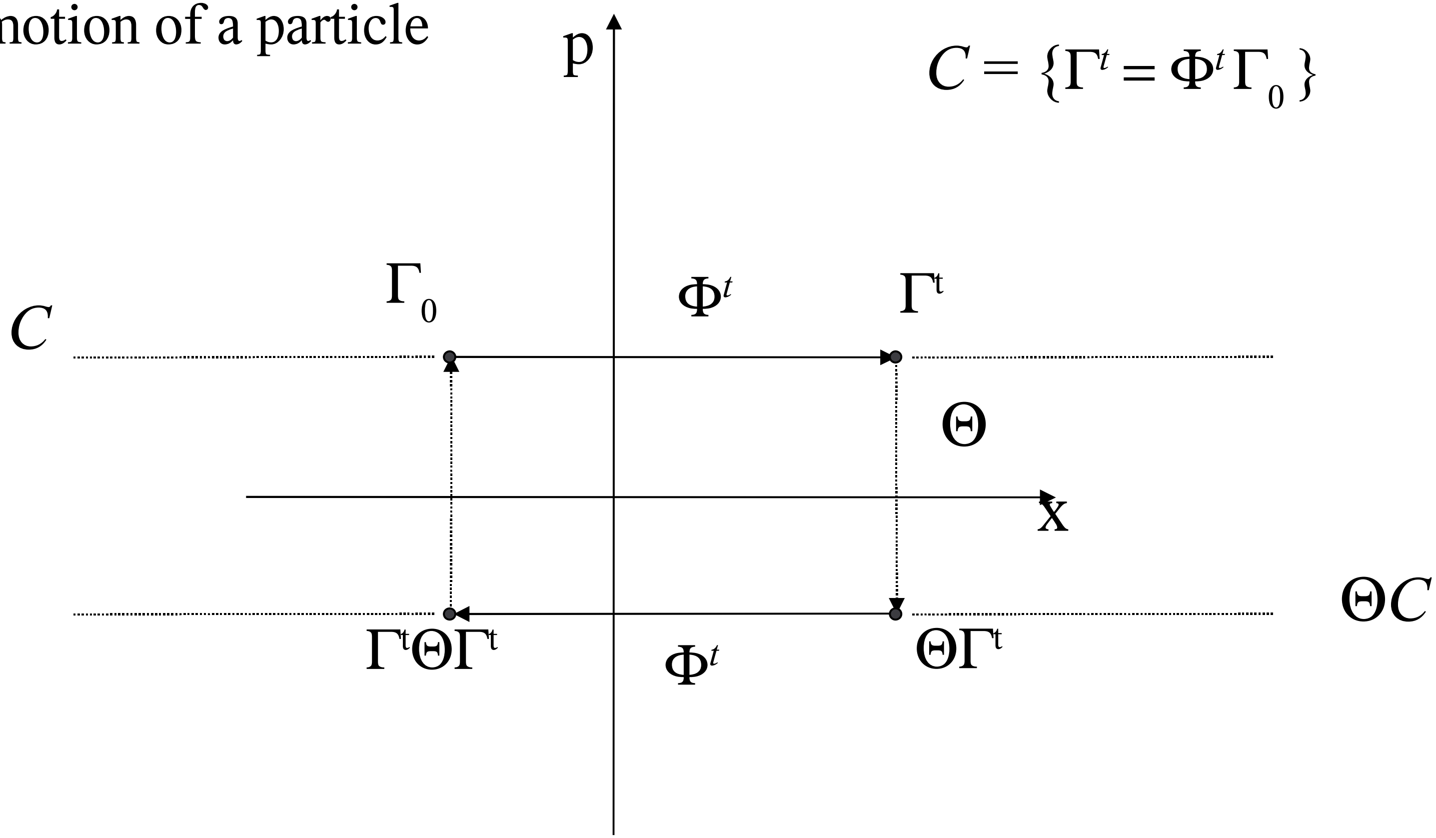

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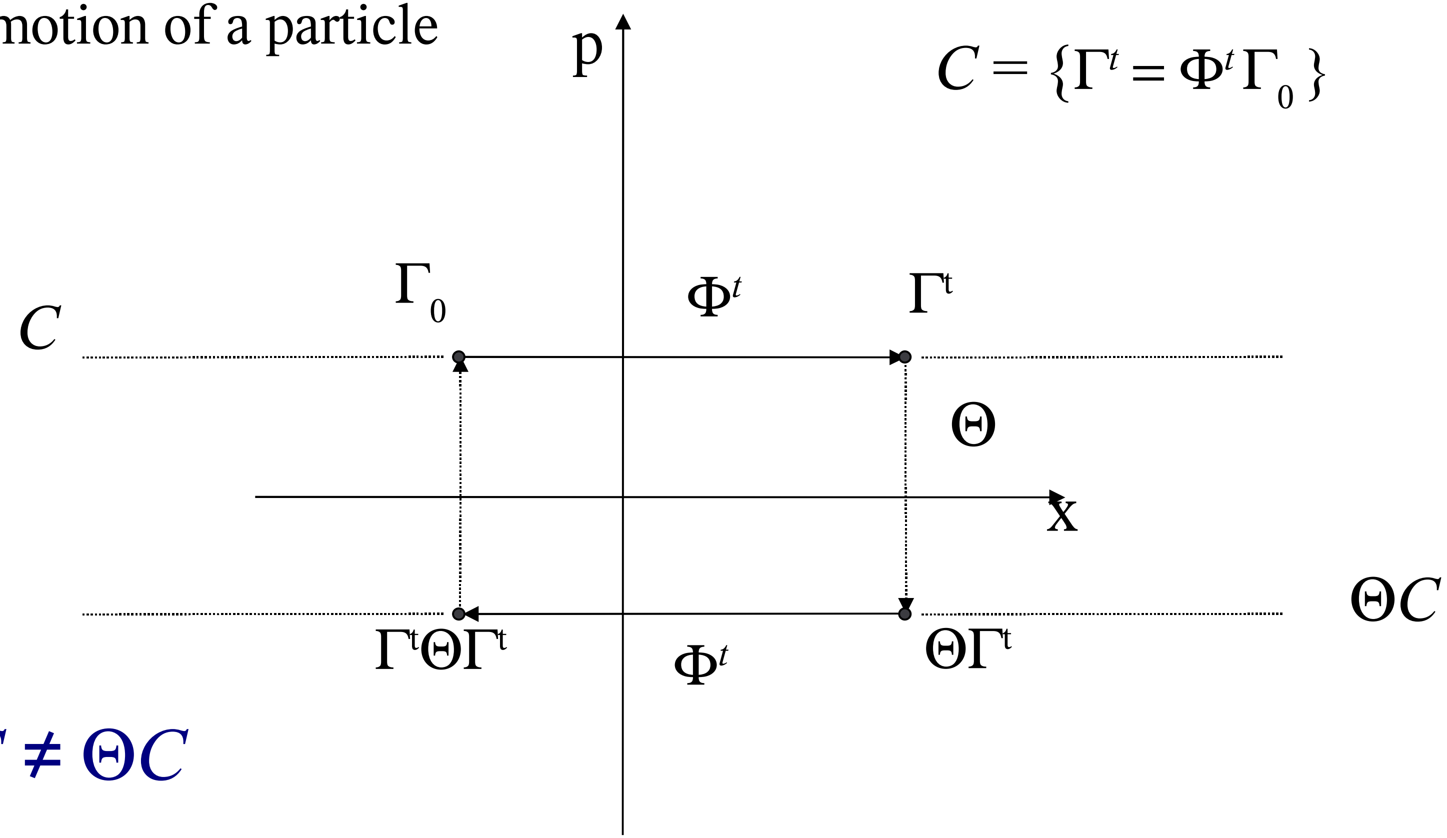

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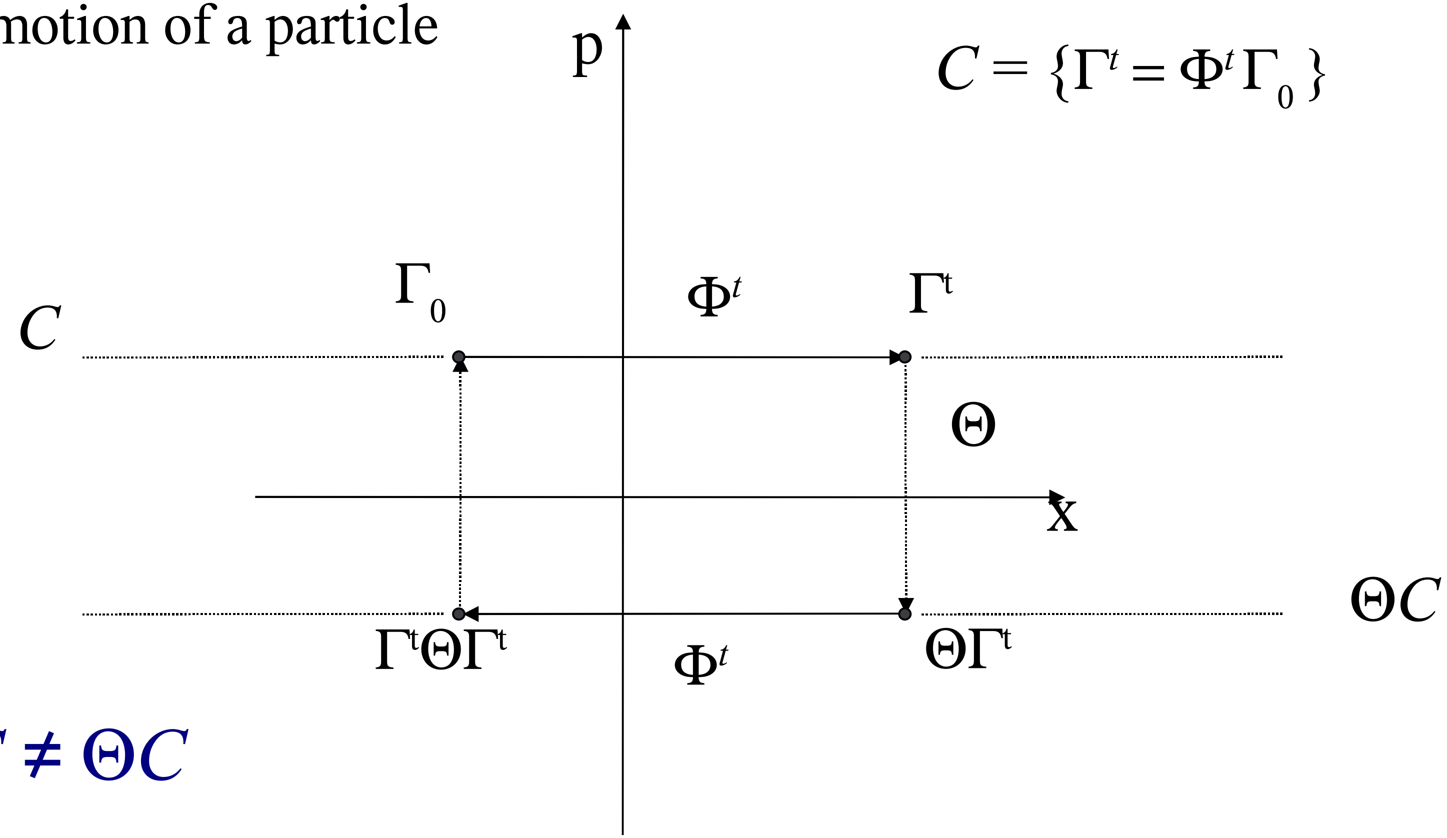

$C \neq \Theta C$

Spontaneous symmetry breaking: the solutions of an equation have a lower symmetry than te equation itself

The time asymmetry results from the *selection* of the trajectories

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Under *nonequilibrium* conditions,

$$\mu(\omega_0\omega_1\cdots\omega_{n-1}) \neq \mu(\omega_{n-1}\cdots\omega_1\omega_0)$$

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Nonequilibrium states are characterized by a positive thermodynamic *entropy production*:

$$\frac{dS}{dt} = \frac{d_{\rm e}S}{dt} + \frac{d_{\rm i}S}{dt} \quad \text{with} \quad \frac{d_{\rm i}S}{dt} \geq 0$$

Nonequilibrium boundary conditions

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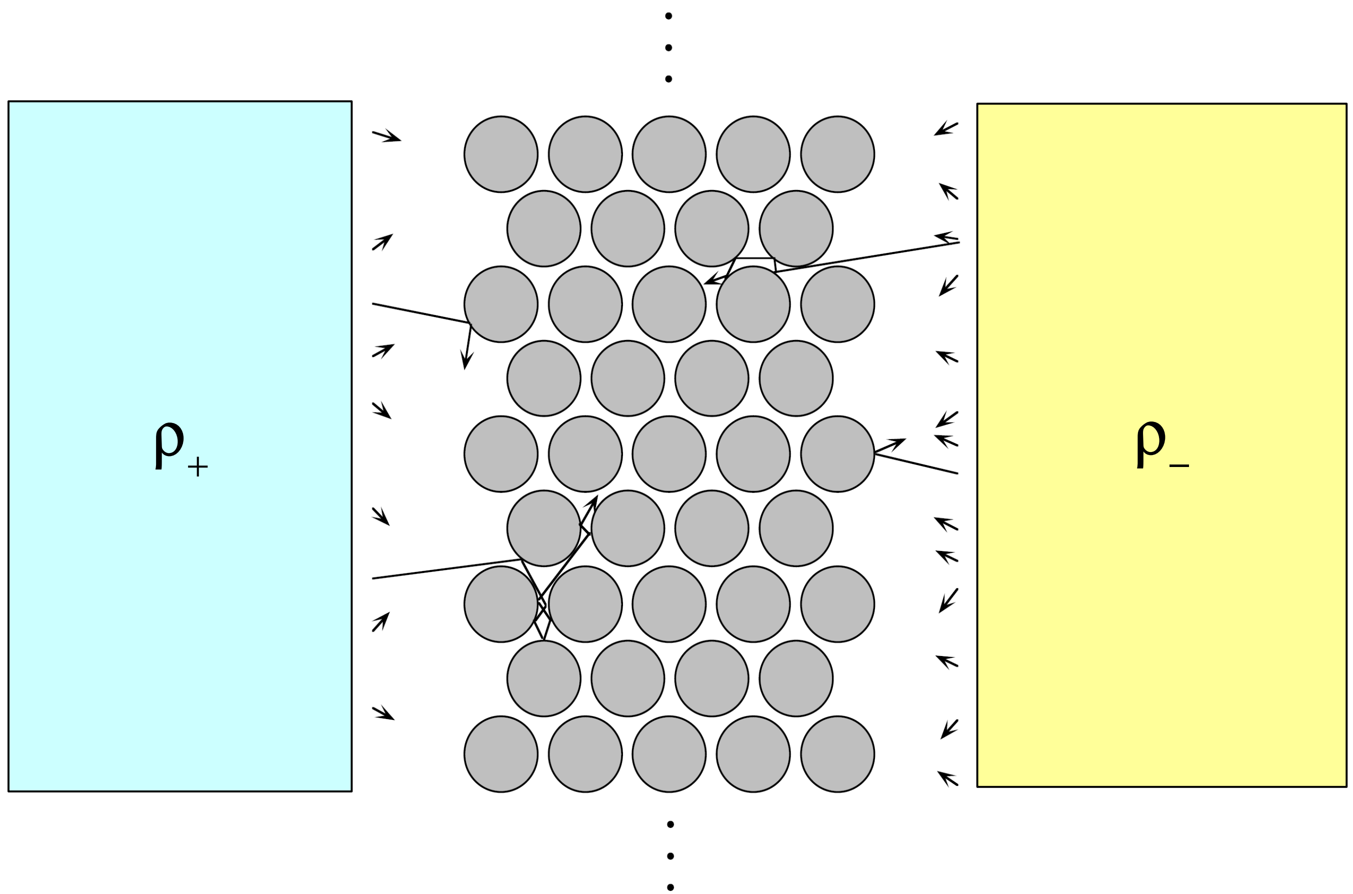

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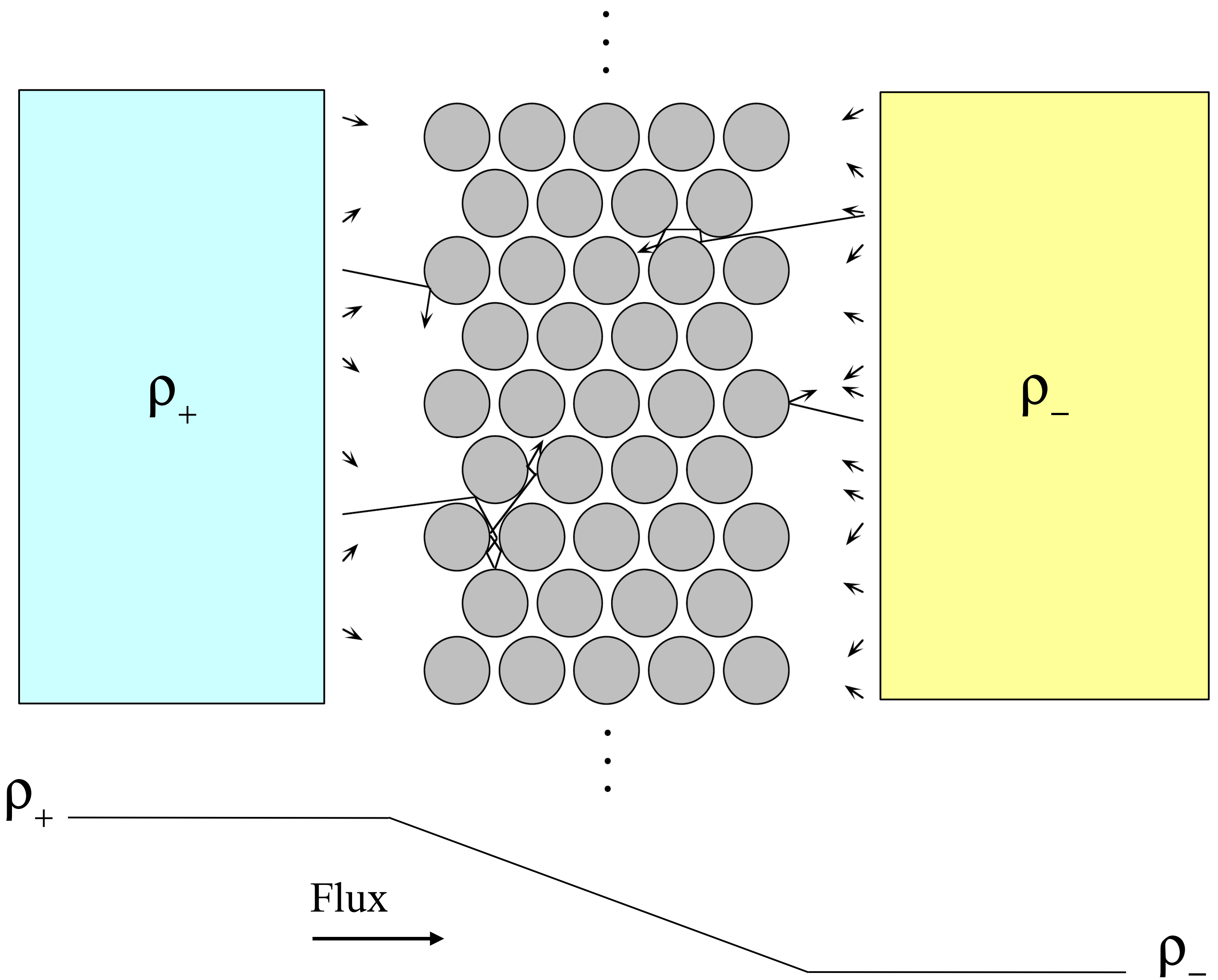
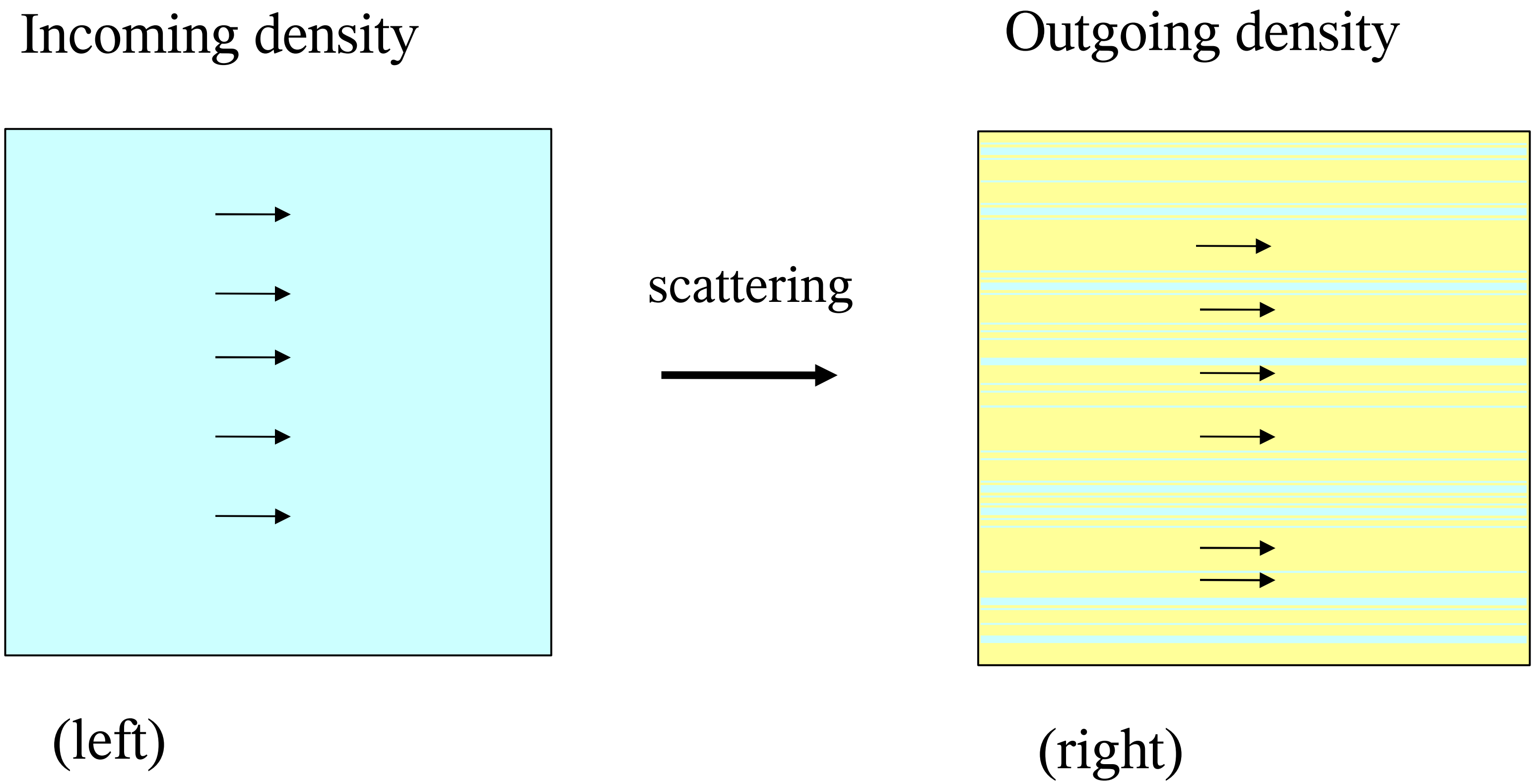
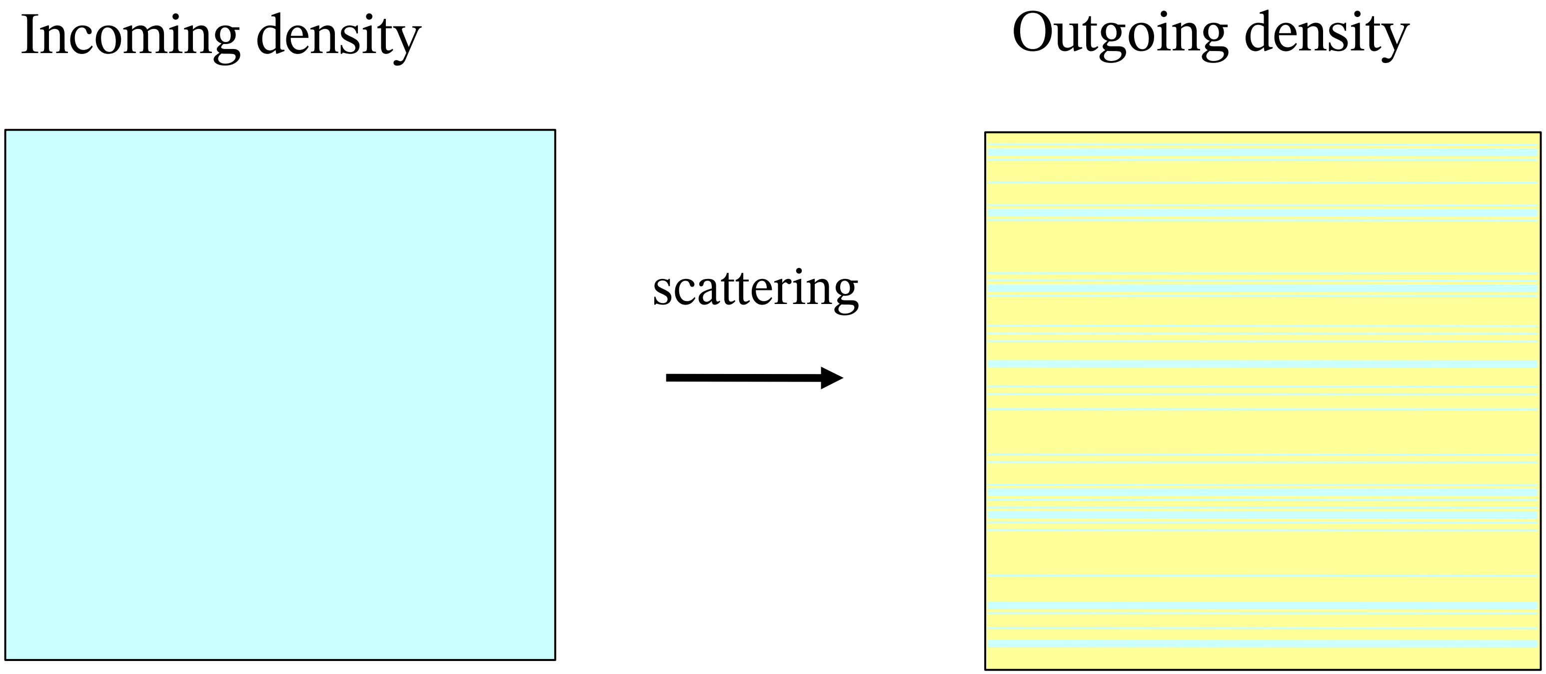
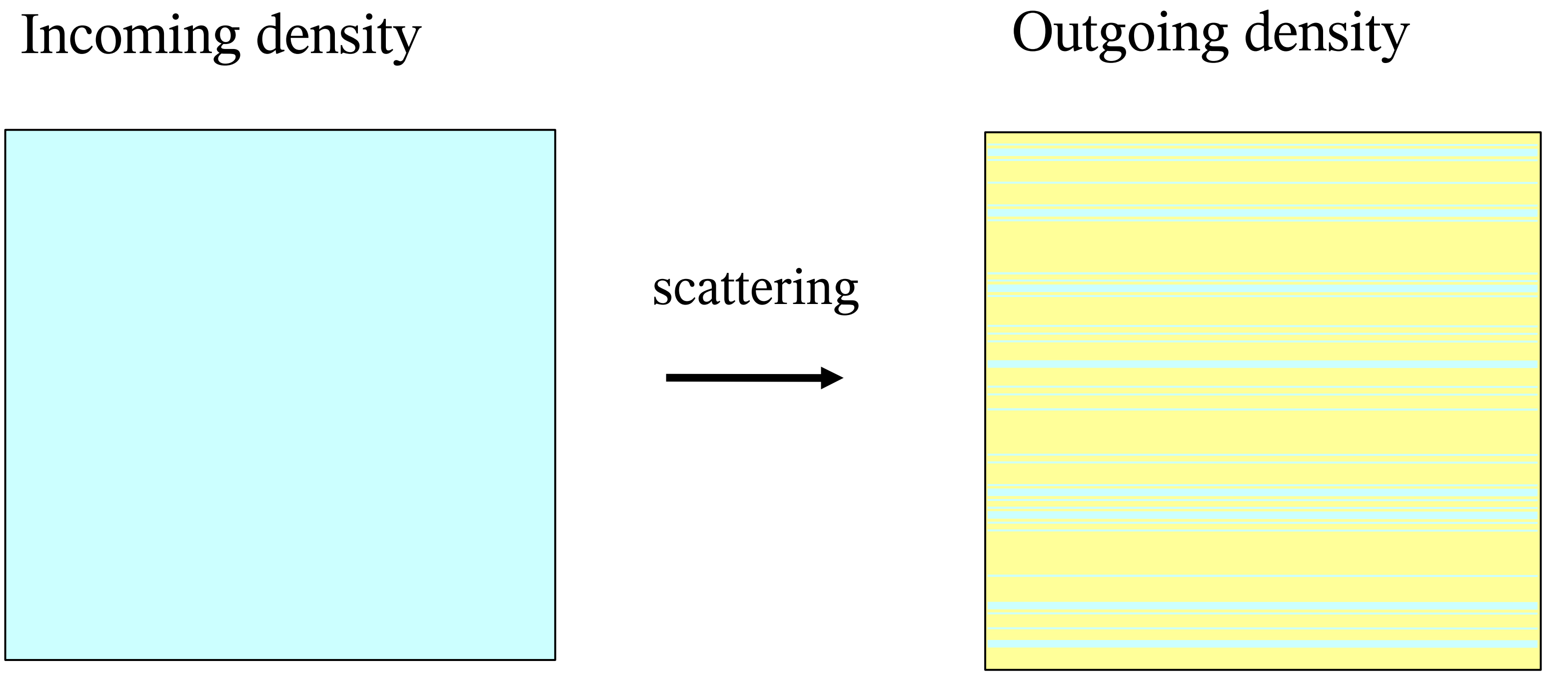



Incoming density
Outgoing density
scattering
(left)
(right)



The time reversal steady state (anti-flux) is possible but particles must be injected with the highly irregular outgoing density



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Fine grained boundary conditions select out a distribution in phase space which is not symmetric under time-reversal

The singular character is of fundamental importance and has been used to construct the *hydrodynamic modes*, which are given by the Policott-Ruelle resonances of the Liouvillian dynamic

P. Gaspard, Phys. Rev. E **53,** 4379 (1996)

P. Gaspard *et al*, Phys. Rev. Lett. **86**, 1506 (2001)

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For such scattering systems, an *ab initio* calculation of the entropy production can be achieved from the stationary measure in phase space

P. Gaspard, J. Stat. Phys. **88**, 1215 (1997)

T. Gilbert, J. R. Dorfman, and P. Gaspard, Phys. Rev. Lett. **85**, 1606 (2000)

Kolmogorov-Sinai entropy per unit time

$$h \equiv \lim_{n\to\infty} -\frac{1}{n\tau} \sum_{\omega_0\omega_1\cdots\omega_{n-1}} \mu(\omega_0\omega_1\cdots\omega_{n-1}) \ln \mu(\omega_0\omega_1\cdots\omega_{n-1})$$

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Shannon-McMillan-Breiman theorem:

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$\longrightarrow$ *h* is the minimal compression of the time series $\omega_0\omega_1\ldots\omega_{n-1}$

$\longrightarrow$ *h* characterizes the *temporal disorder* in the dynamical evolution

Birth and death processes

$$h = \Big(\sum_{\omega'} p_\omega W_{\omega\omega'} \Big) \ln(e/\tau) - \sum_{\omega'} p_\omega W_{\omega\omega'} \ln W_{\omega\omega'} + O(\tau)$$

where $W_{\omega\omega'}$ = transition rates

p_ω = stationary probabilities

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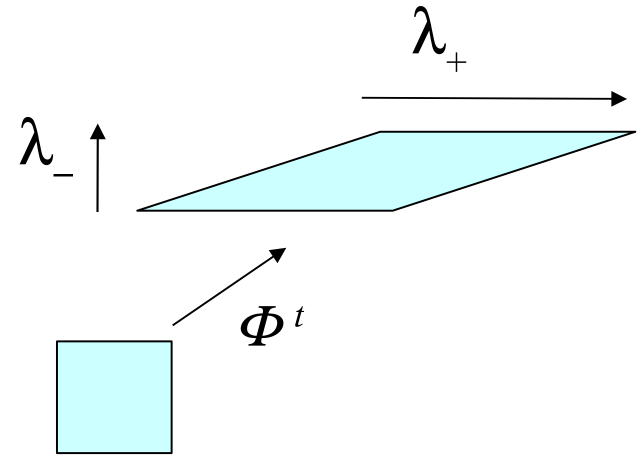
The divergence $h(\tau) \sim \ln(e/\tau)$ for τ going to zero

characterizes the randomness of the process and arises from

our assumption of a stochastic behavior *at all scales*

Chaotic dynamical systems

Pesin's theorem: $$h_{KS} = \sum_{i>0} \lambda_i$$



where the λ's are the *Lyapounov exponents*:

(h is defined as the supremum over all possible partitions)

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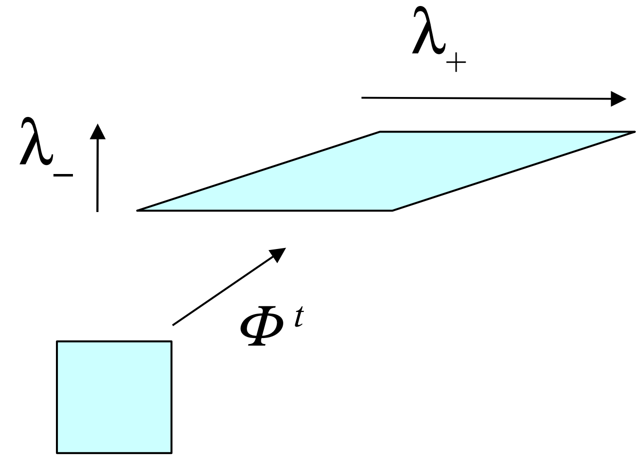

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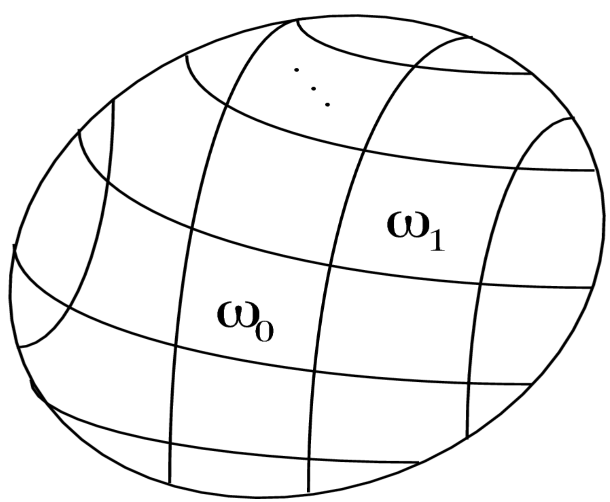

→ h = *information creation rate* of the dynamics

KS entropy is a microscopic quantity which describes the fine dynamical correlations.

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For example, for a gas a room temperature, the Lyapounov exponent can be estimated as

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The KS entropy thus captures the features of the dynamic on the *microscopic time scales* characteristic of the process

Time-reversed entropy per unit time

$$h^R \equiv \lim_{n\to\infty} -\frac{1}{n\tau} \sum_{\omega_0\omega_1\cdots\omega_{n-1}} \mu(\omega_0\omega_1\cdots\omega_{n-1}) \ln \bar{\mu}(\omega_{n-1}\cdots\omega_1\omega_0)$$

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$\longrightarrow$ h^{R} characterizes the *temporal disorder* of the time-reversed process

Properties

- $h^R = h \quad \Longleftrightarrow \quad \mu(\omega_0\omega_1\cdots\omega_{n-1}) = \bar{\mu}(\omega_{n-1}\cdots\omega_1\omega_0)$

  $\Longleftrightarrow$ Equilibrium

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Temporal ordering principle :

In nonequilibirum steady states, the typical paths are more ordered in time than their time-reversed counterparts.

The entropies h and h^{R} measure the time-symmetry breaking under nonequilibrium conditions.

The *thermodynamic entropy production* is related to the difference of these entropies as

$$\frac{d_i S}{dt} = h^R - h$$

P. Gaspard, J. Stat. Phys. **117**, 599 (2004)

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The entropy production is here expressed in terms of two *microscopic* quantities measuring the *time-reversal symmetry breaking* at the level of *dynamical randomness*

P. Gaspard, J. Stat. Phys. **117**, 599 (2007)

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while their difference gives

$$\frac{d_i S}{dt} = h^R - h = \sum_{\omega'} p_\omega W_{\omega\omega'} \ln \frac{W_{\omega\omega'}}{W_{\omega'\omega}}$$

which is the well-known entropy production

Nonequilibrium chaotic systems

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At equilibrium, $h^R = h$ because of Liouville's theorem

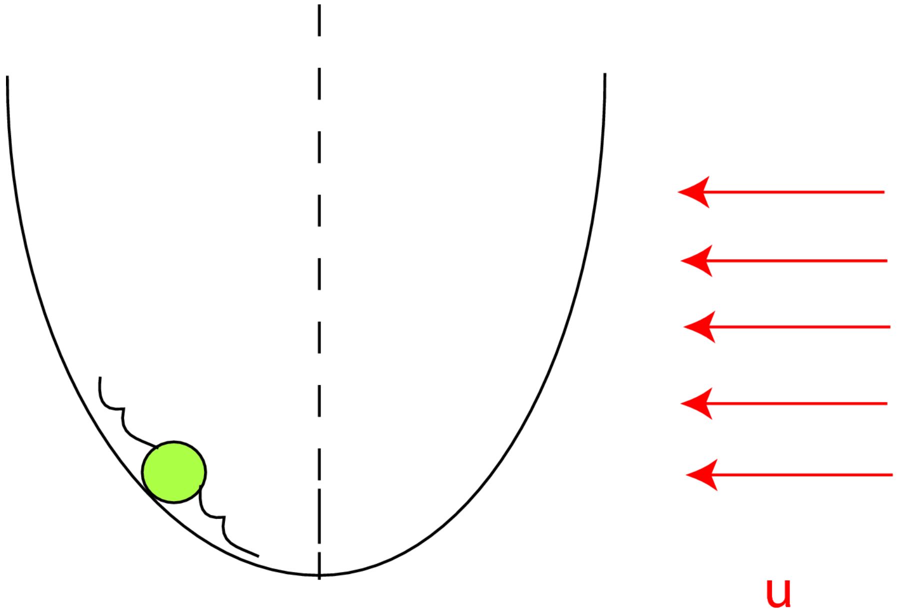
Experimental results

- Driven Brownian motion
- Driven *RC* circuit

Phys. Rev. Lett. **98**, 150601 (2007)

arXiv:cond-mat/0710.3646 (2007)

Driven Brownian Motion (by S. Ciliberto and A. Petrosyan)



Viscous fluid moving at velocity u

and friction coefficient α

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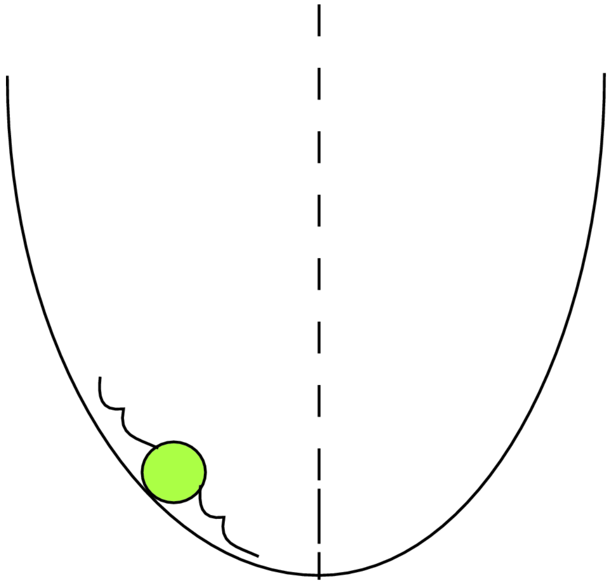

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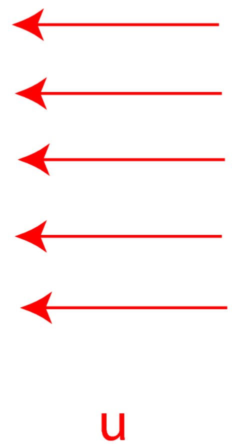


$$\alpha \frac{dz}{dt} = F(z) + \alpha u + \xi_t$$

with random noise:

$$\begin{aligned} \langle \xi_t \rangle &= 0 \\ \langle \xi_t \, \xi_{t'} \rangle &= 2 \, k_{\mathrm{B}} T \, \alpha \, \delta(t - t') \end{aligned}$$

At long times, the system reaches a *nonequilibrium stationary state.*

For an harmonic potential, $V = \frac{1}{2} k z^2$,

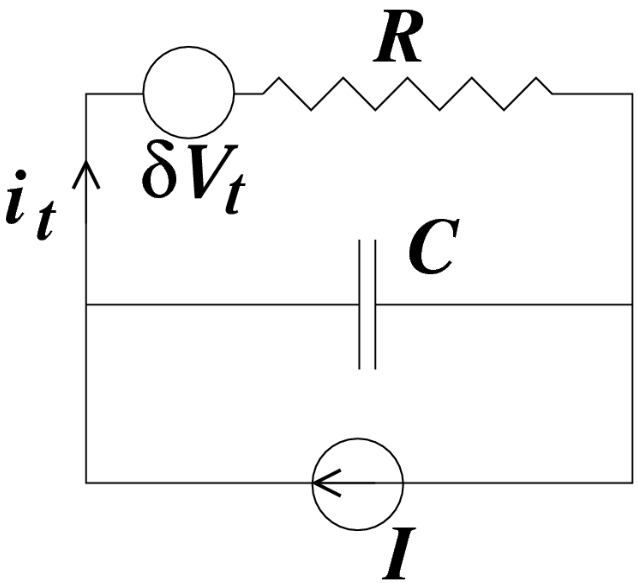
the stationary distribution is Gaussian

$$p_{\rm st}(z) = \sqrt{\frac{\beta k}{2\pi}} \exp\left[-\frac{\beta k}{2}(z - u\,\tau_R)^2 \right] ,$$

with the relaxation time $\tau_R = \frac{\alpha}{k}$.

Experimentally, $k = 9.62$ μkg/s^{-2} while $\tau_R = 3.05\ 10^{-3}$ s

Equivalent to a *RC* circuit driven by a current source
(by N. Garnier and S. Joubaud)



Brownian particle	RC circuit
z_t	$q_t - It$
$\dot{z}_t$	$\dot{q}_t - I$
ξ_t	$-\delta V_t$
α	R
k	$1/C$
u	$-I$

R= 9,22 MΩ
C= 278 pF

I = driving force

Stochastic energetics

Heat: $$Q_t = \int_0^t (\dot{z}_{t'} - u)\; F(z_{t'})\, dt'$$

$$= V(z_0) - V(z_t) - u \int_0^t\; dt'\; F(z)$$

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Entropy production in the NESS:

$$\frac{d_{\mathrm{i}} S}{dt} = \lim_{t\to\infty} \frac{1}{t} \frac{\langle Q_t \rangle}{T} = \frac{\alpha u^2}{T} \quad \left(= \frac{RI^2}{T} \right)$$

From a phase space perspective, the probability densities are given by an Onsager-Machlup functional

$$P[z_t|z_0] \propto \exp\left[-\frac{1}{4k_{\mathrm{B}}T\alpha}\int_0^t dt'\,\Big(\alpha\dot{z} - F(z) - \alpha u\Big)^2\right]$$

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The dissipation can be obtained by considering the time-reversal paths *and* by reversing the speed *u* (odd driving constraints), so that

$$\ln\frac{P_+[z_t|z_0]}{P_-[z_t^\mathrm{R}|z_0^\mathrm{R}]} = \frac{1}{k_\mathrm{B}T}\left[V(z_0) - V(z_t) - u\int_0^t dt'\,F(z)\right]$$

The thermodynamic entropy production is thus expressed as

$$\frac{d_{\rm i}S}{dt} = \lim_{t\to\infty} \frac{k_{\rm B}}{t} \int \mathcal{D}z_t \, P_+[z_t] \, \ln \frac{P_+[z_t]}{P_-[z_t^{\rm R}]} \geq 0$$

$$= \frac{\alpha u^2}{T}$$

in terms of the joint probabilities $P[z_t] \propto P[z_t|z_0] \, p_{\rm st}(z_0)$.

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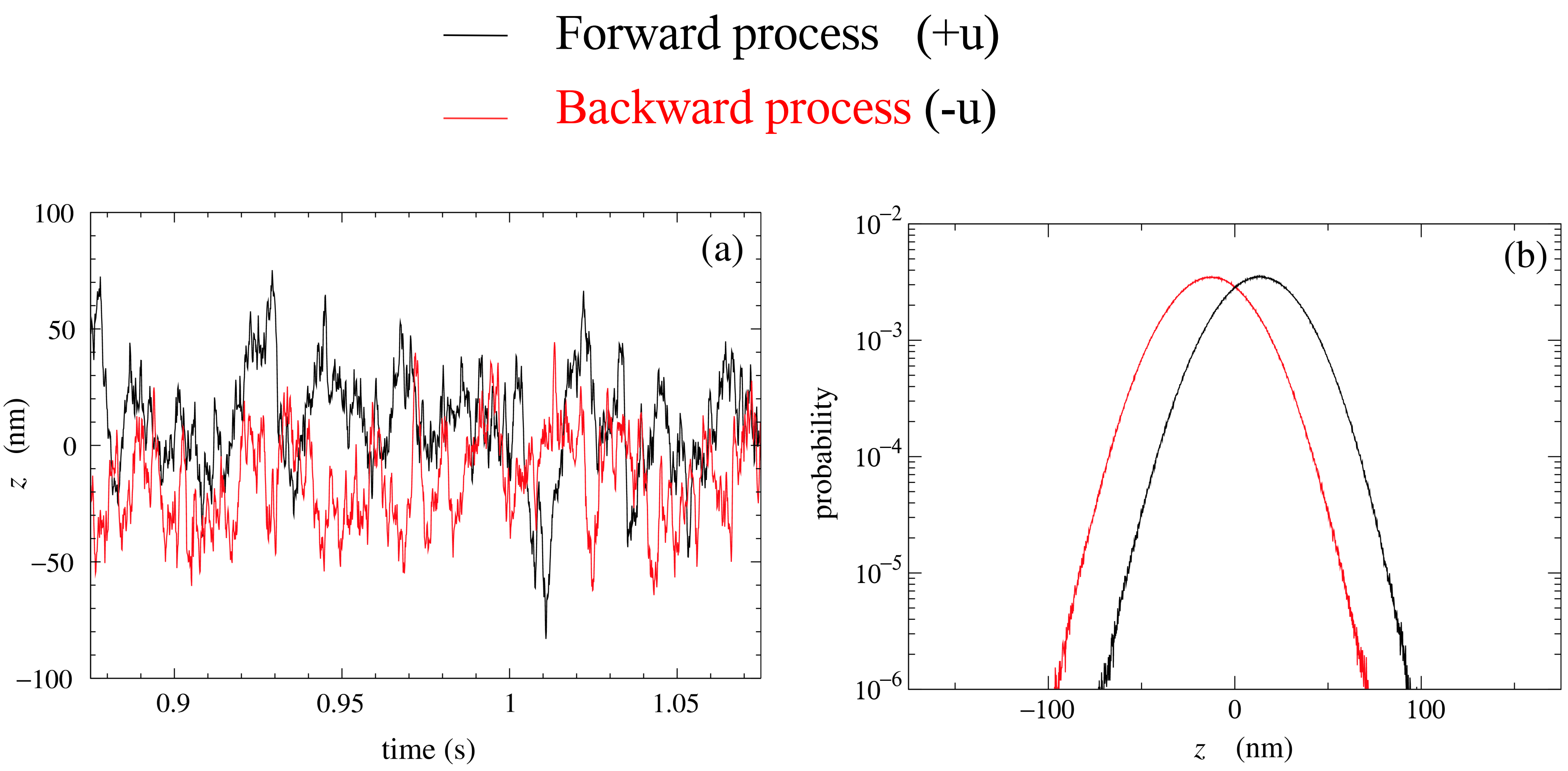
At equilibrium,

$$P_+[z_t] = P_-[z_t^{\rm R}] \qquad (u{=}0)$$

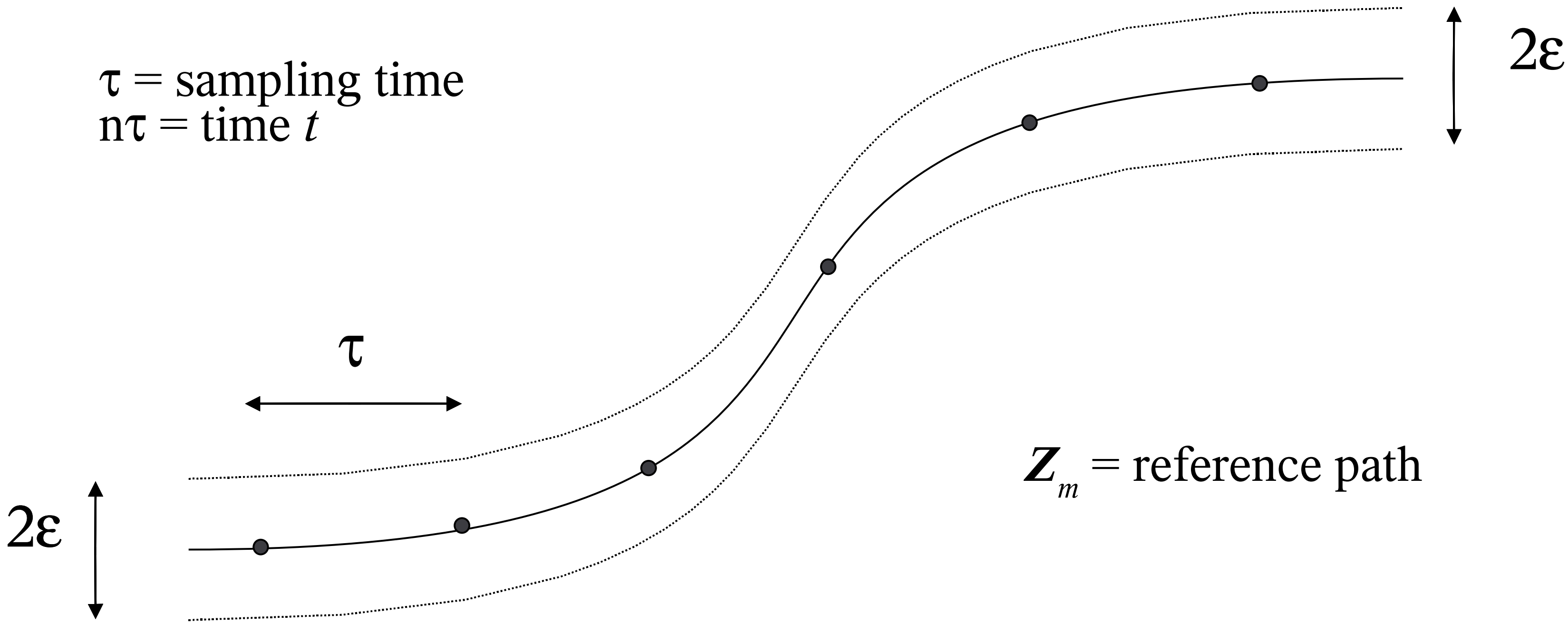
and the entropy production vanishes.

Stationary state of the Brownian particle



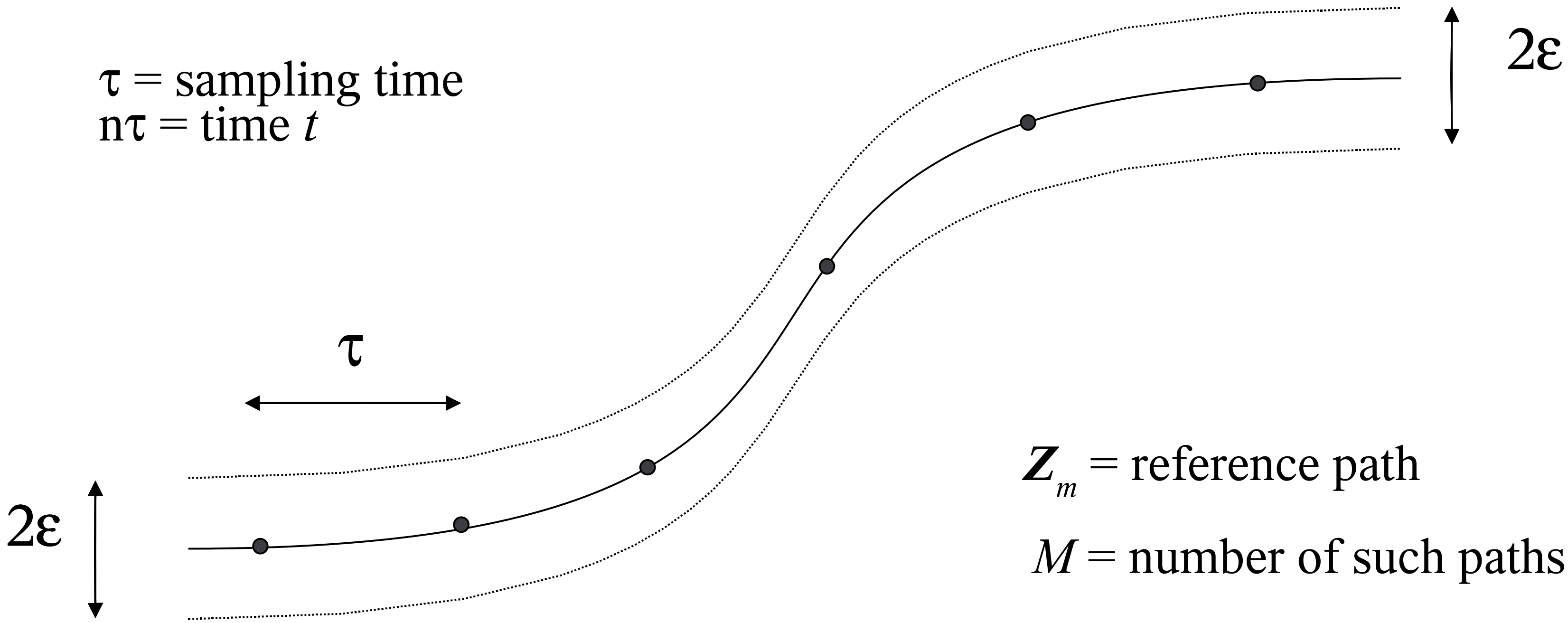
(u = 4.24 μm/s)

How can we obtain the entropies per unit time?



How can we obtain the entropies per unit time?

τ = sampling time
$n\tau$ = time t



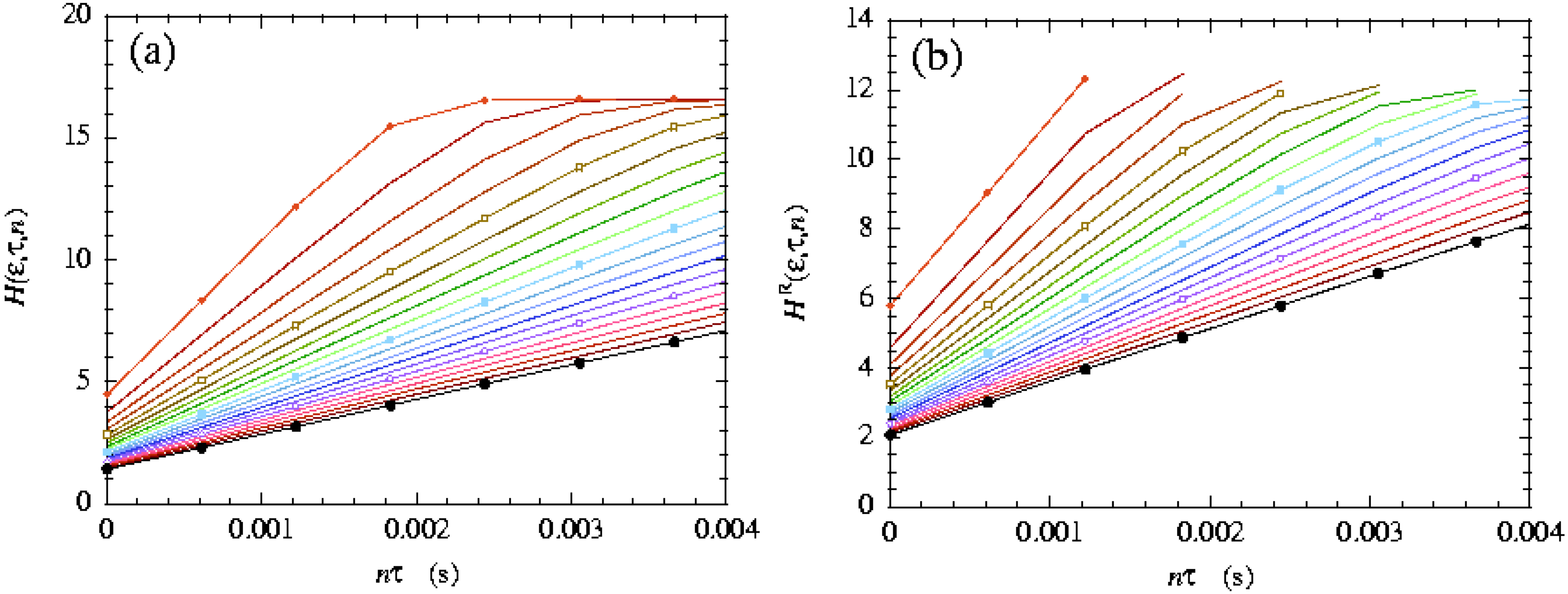
$\boldsymbol{Z}_m$ = reference path

M = number of such paths

$$H(\varepsilon,\tau,n) = -\frac{1}{M}\sum_{m=1}^{M} \ln P_{+}(\boldsymbol{Z}_m;\varepsilon,\tau,n)$$

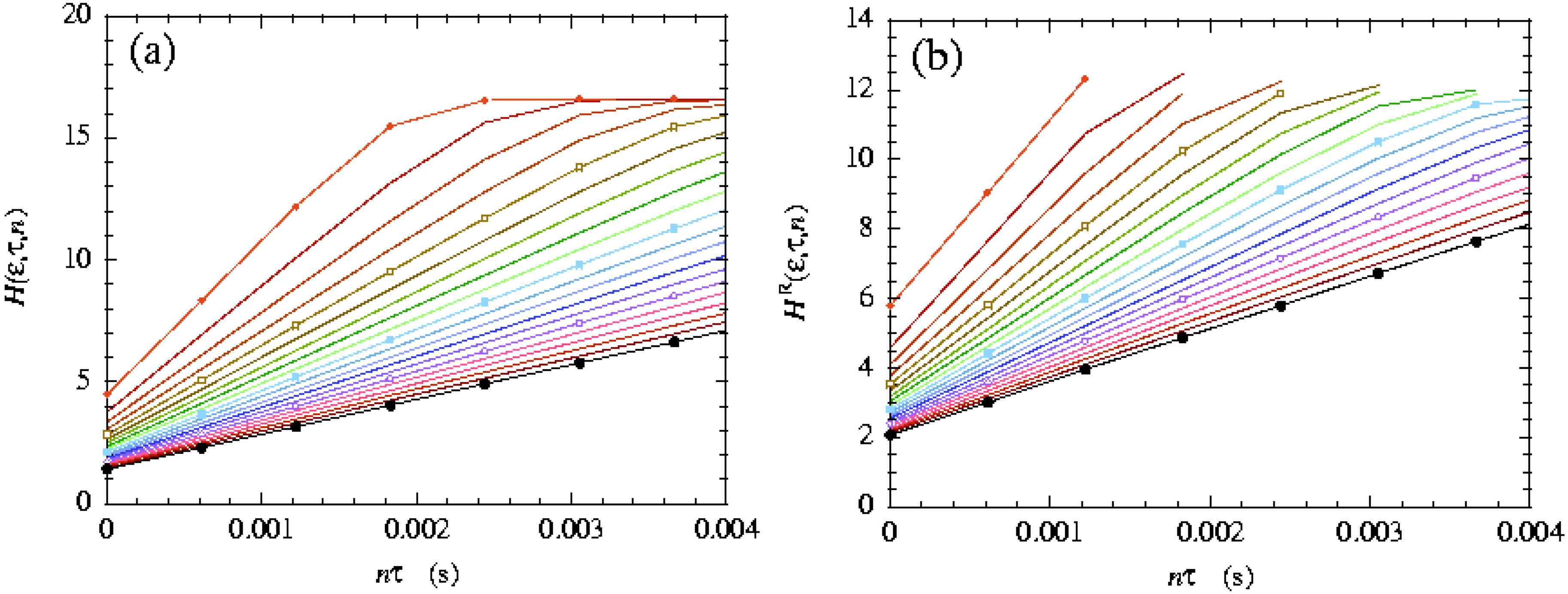
« Pattern entropies »

$$H^{\mathrm{R}}(\varepsilon,\tau,n) = -\frac{1}{M}\sum_{m=1}^{M} \ln P_{-}(\boldsymbol{Z}_m^{\mathrm{R}};\varepsilon,\tau,n)$$



Increasing slopes for smaller values of ε

(ε between 0.558 - 11.16 nm)



The linear growth of these pattern entropies defines

$$h(\varepsilon,\tau) = \lim_{n\to\infty} \lim_{L'\to\infty} \frac{1}{\tau}\Big[H(\varepsilon,\tau,n+1) - H(\varepsilon,\tau,n)\Big]$$

$$h^{\mathrm{R}}(\varepsilon,\tau) = \lim_{n\to\infty} \lim_{L'\to\infty} \frac{1}{\tau}\Big[H^{\mathrm{R}}(\varepsilon,\tau,n+1) - H^{\mathrm{R}}(\varepsilon,\tau,n)\Big]$$

Analytical result: $$h(\varepsilon, \tau) = \frac{1}{\tau} \ln \sqrt{\frac{\pi \mathrm{e} D \tau_R}{2\varepsilon^2} \left(1 - \mathrm{e}^{-2\tau/\tau_R}\right)} + O(\varepsilon^2)$$

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with the scaled variable $\delta \equiv \varepsilon / \sqrt{1 - \exp(-2\tau/\tau_R)}$

it becomes $$\tau h = \ln \sqrt{\frac{\pi e D \tau_R}{\delta^2}}$$

which is independent of the sampling time

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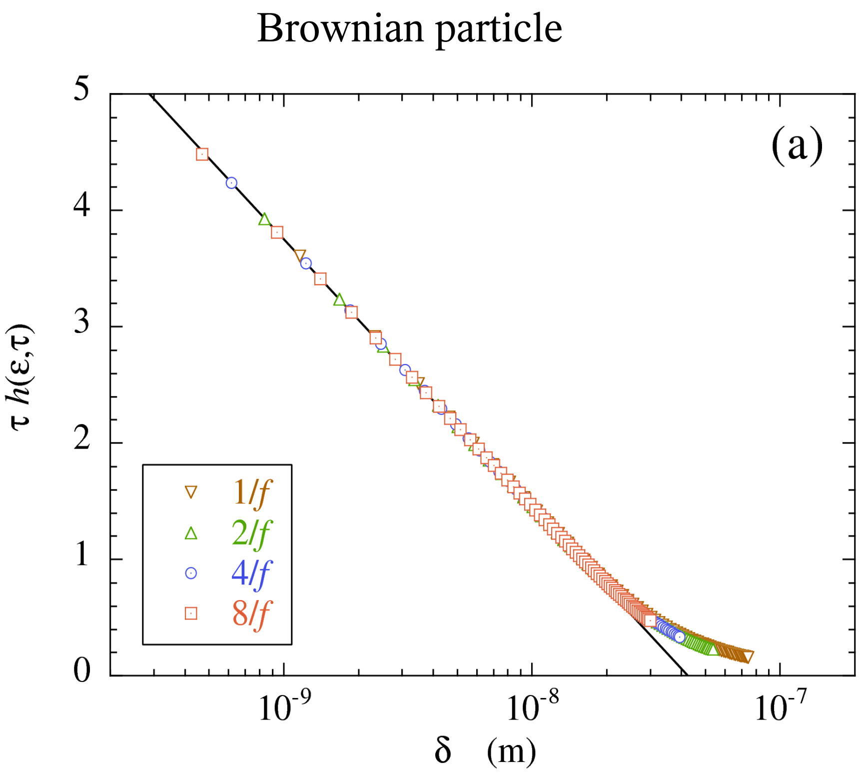
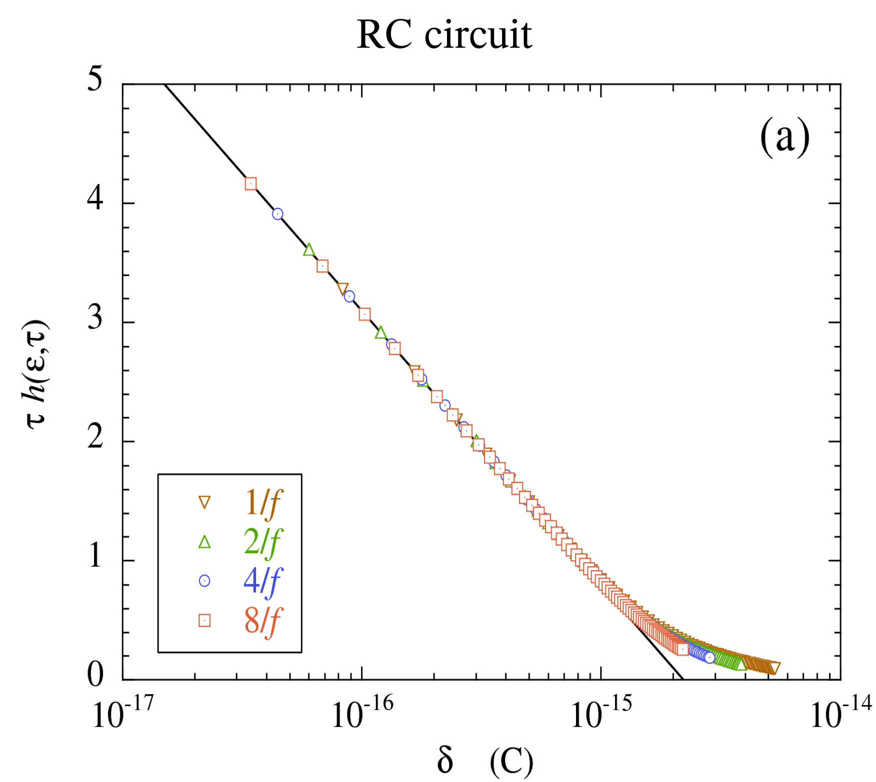
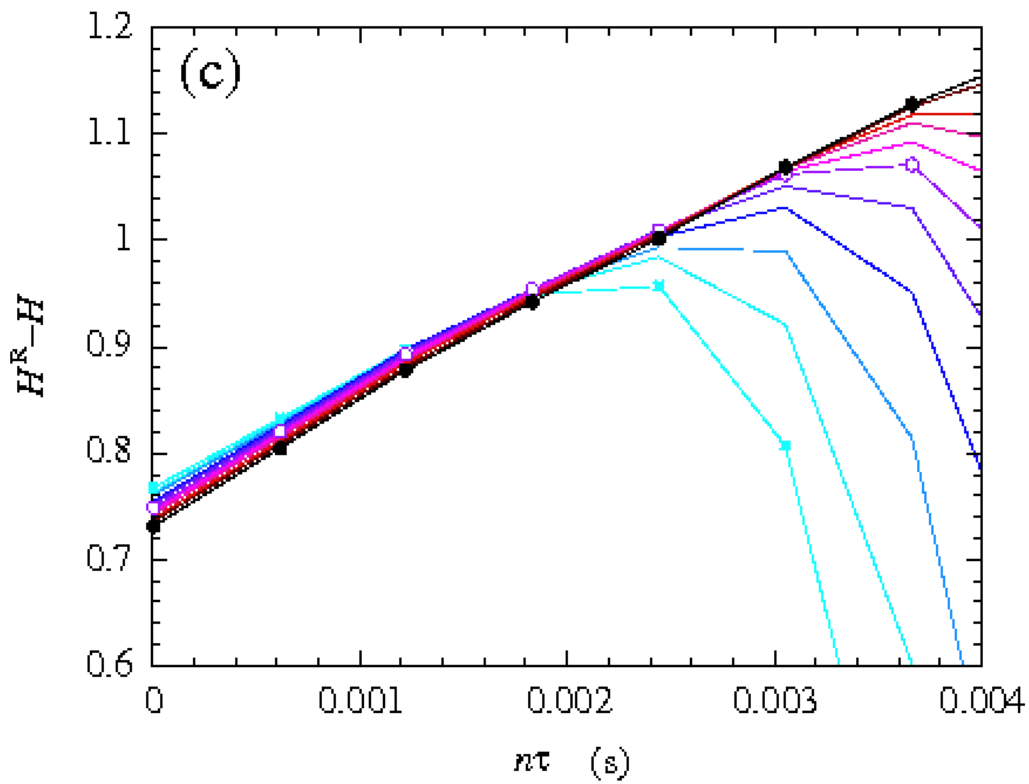




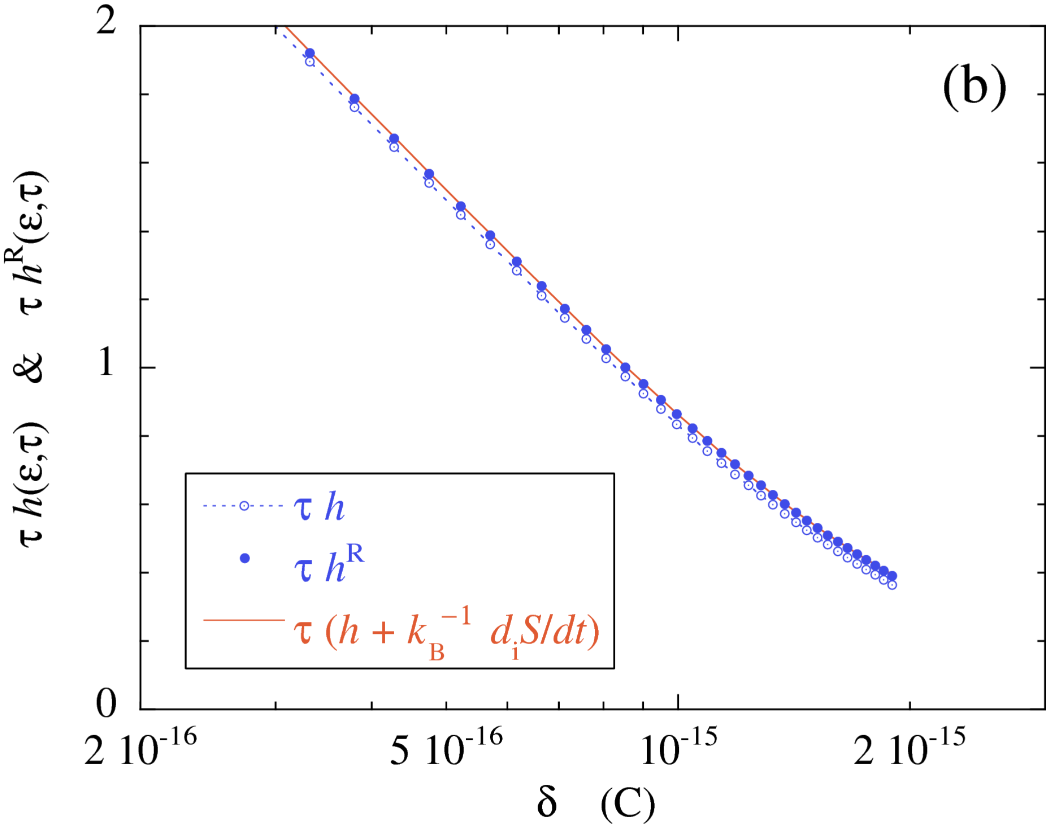
$$\delta \equiv \varepsilon/\sqrt{1-\exp(-2\tau/\tau_R)}, \qquad \tau h = \ln\sqrt{\frac{\pi e D\tau_R}{\delta^2}}$$

Difference between the pattern entropies H^{R} and H



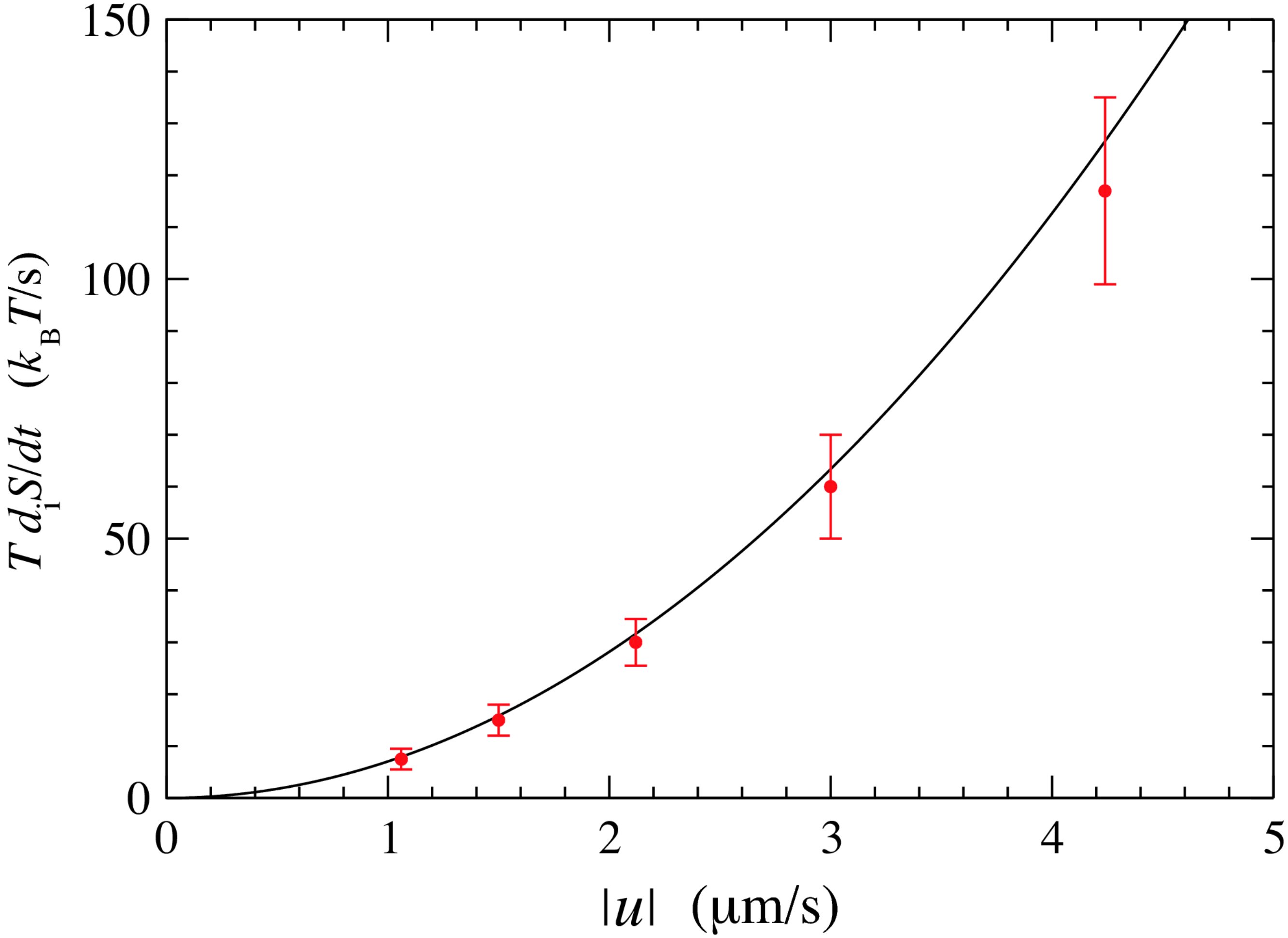
$$\frac{d_{\mathrm{i}}S}{dt} = \lim_{\varepsilon\to 0}\lim_{\tau\to 0}\ k_{\mathrm{B}}\left[h^{\mathrm{R}}(\varepsilon,\tau) - h(\varepsilon,\tau)\right]$$

Brownian particle ($u = 4.24$ μm/s)

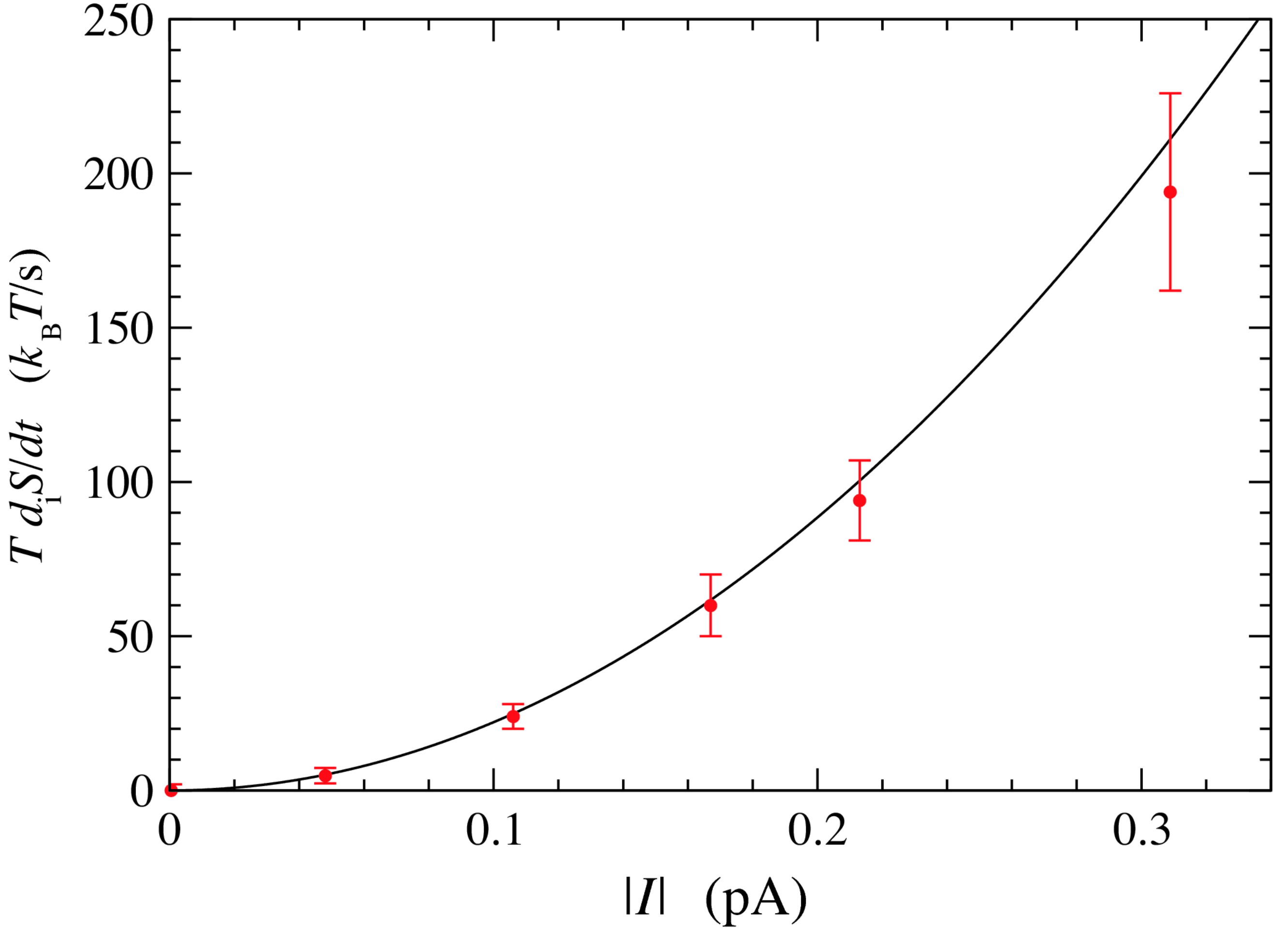


$$\tau h^{\mathrm{R}} \simeq \tau \left(h + k_{\mathrm{B}}^{-1} d_{\mathrm{i}} S / dt \right)$$

Entropy production for the Brownian particle



Entropy production for the RC circuit

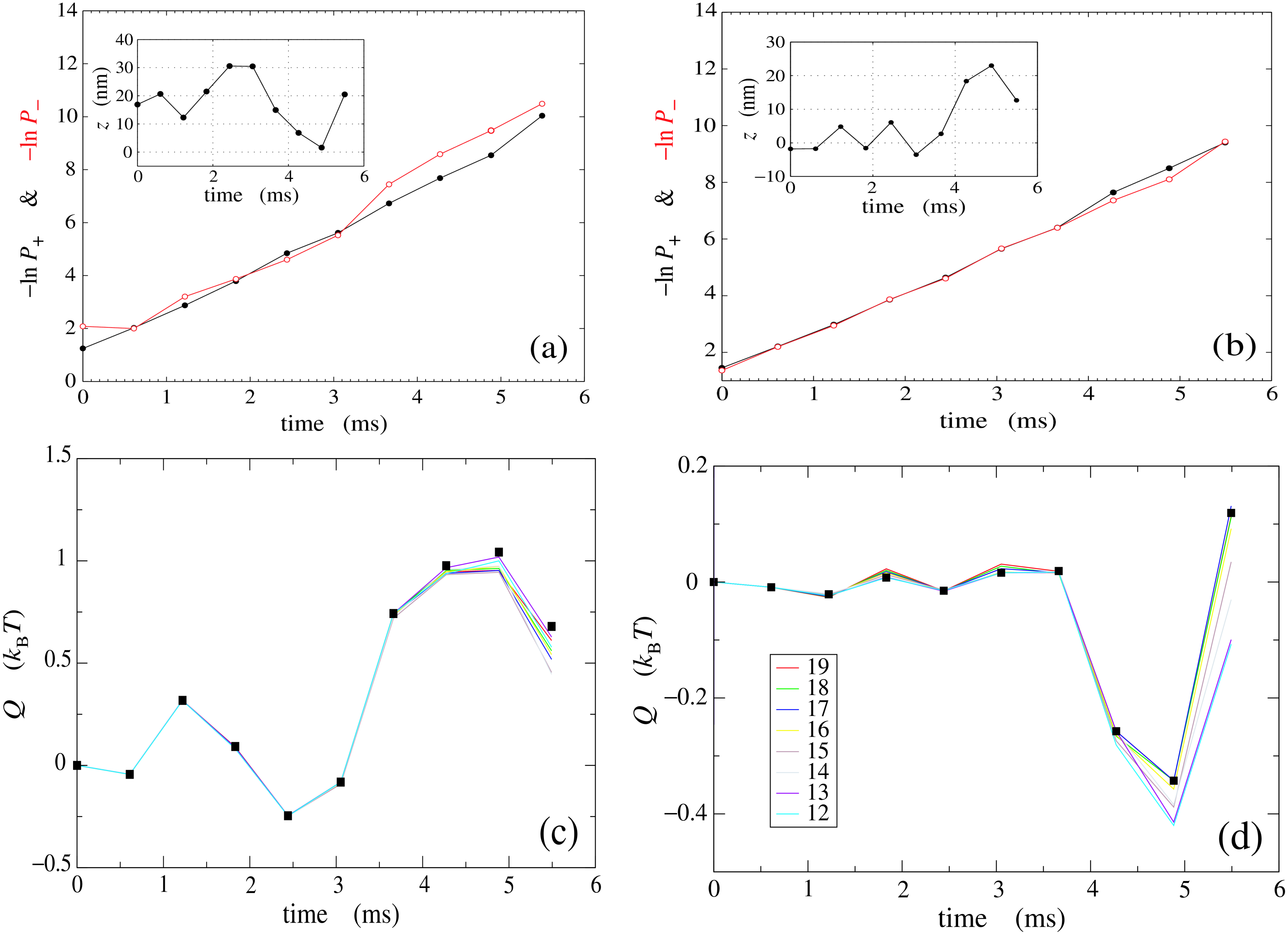


Trajectory picture:

The heat dissipated along random trajectories is obtained from the phase space probabilities as

$$\ln \frac{P_+[z_t|z_0]}{P_-[z_t^{\mathrm{R}}|z_0^{\mathrm{R}}]} = \frac{1}{k_{\mathrm{B}}T}\left[V(z_0) - V(z_t) - u\int_0^t dt'\, F(z)\right]$$

$$= Q_t$$



−ln P+ & −ln P−
time (ms)
z (nm)
(a)
(b)
Q (kBT)
(c)
(d)
19
18
17
16
15
14
13
12

Remarks

Let $\zeta \simeq \frac{1}{t}\frac{Q_t}{T}$ and $J(\zeta) \equiv \lim_{t\to\infty} -\frac{1}{t} \ln \mathrm{Pr}\left\{\zeta < \frac{Q_t}{tT} < \zeta + d\zeta\right\}$

The fluctuation theorem reads

$$\zeta = k_{\mathrm{B}}\Big[J(-\zeta) - J(\zeta)\Big] \qquad \text{for} \qquad -\langle\zeta\rangle \leq \zeta \leq \langle\zeta\rangle$$

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The average dissipation is expressed from the FT as

$$\frac{d_{\mathrm{i}}S}{dt} = k_{\mathrm{B}}J\big(-\langle\zeta\rangle\big) = h^{\mathrm{R}} - h$$

Since $h \geq 0$, we have $h^{\mathrm{R}}(\varepsilon,\tau) \geq J(-\langle\zeta\rangle)$ which shows that the entropies h and h^{R} probe finer scale in phase space where the time-asymmetry is tested

Physics of information : Landauer's principle

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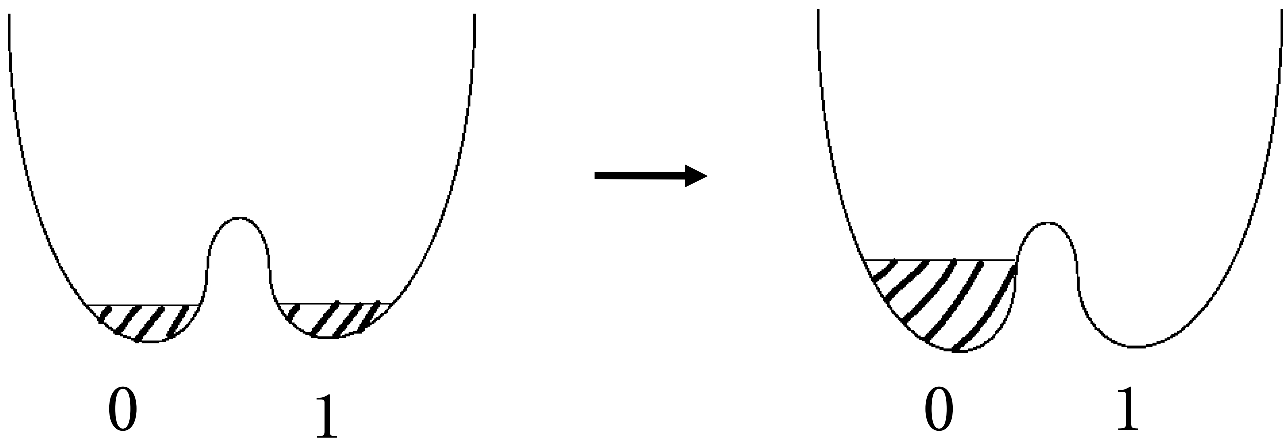
Erasure of random bits in a bistable potential

Physics of information : Landauer's principle

Erasure of random bits in a bistable potential



Change in entropy $\Delta S = \ln 2$ during the erasure must be dissipated as $kT \ln 2$ of heat in the environment

Physics of information : Landauer's principle

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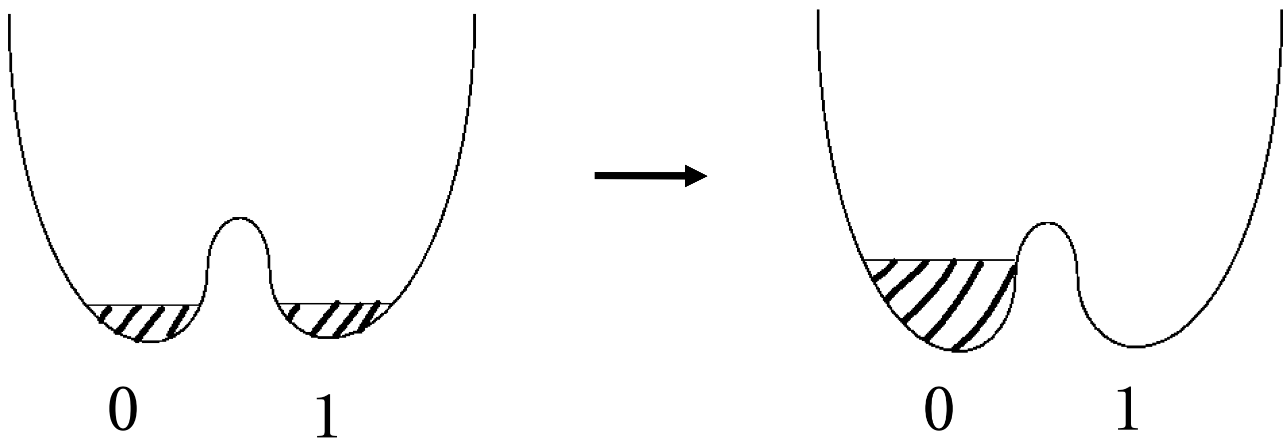

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→ *Minimal dissipation for random bits = kT ln 2 per bit*

Erasure of correlated random bits

...0000111010111101101010111011011...

with probability $p(\sigma_1\sigma_2\cdots\sigma_m)$ to observe the sequence

... $\sigma_1\sigma_2\cdots\sigma_m$ ...

Erasure of correlated random bits

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with probability $p(\sigma_1\sigma_2\cdots\sigma_m)$ to observe the sequence

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The redundancy of the sequence is characterized by

$$I = \lim_{m\to\infty} -\frac{1}{m} \sum_{\sigma_1\sigma_2\cdots\sigma_m} p(\sigma_1\sigma_2\cdots\sigma_m)\ln p(\sigma_1\sigma_2\cdots\sigma_m)$$

which is also its information content ($I \leq \ln 2$)

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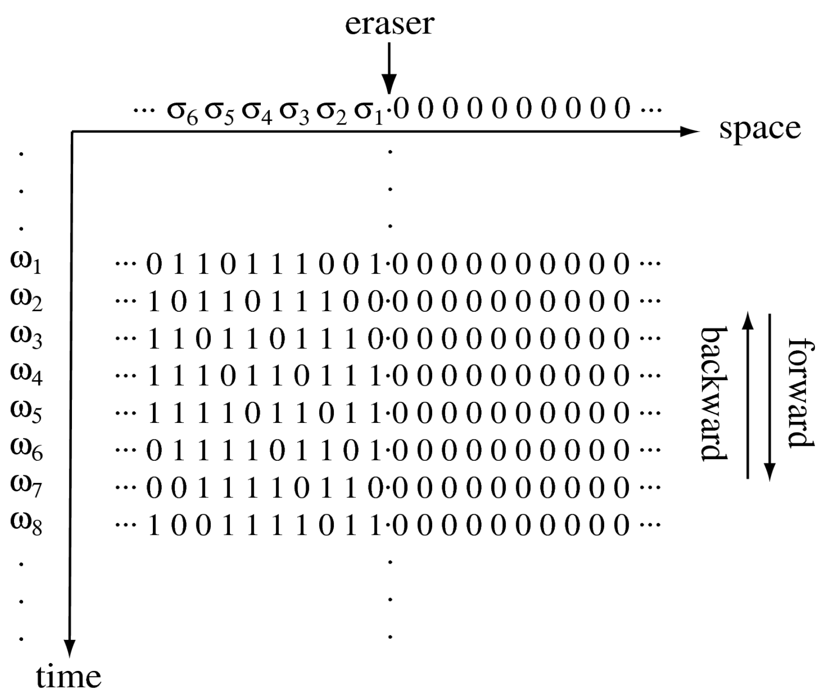
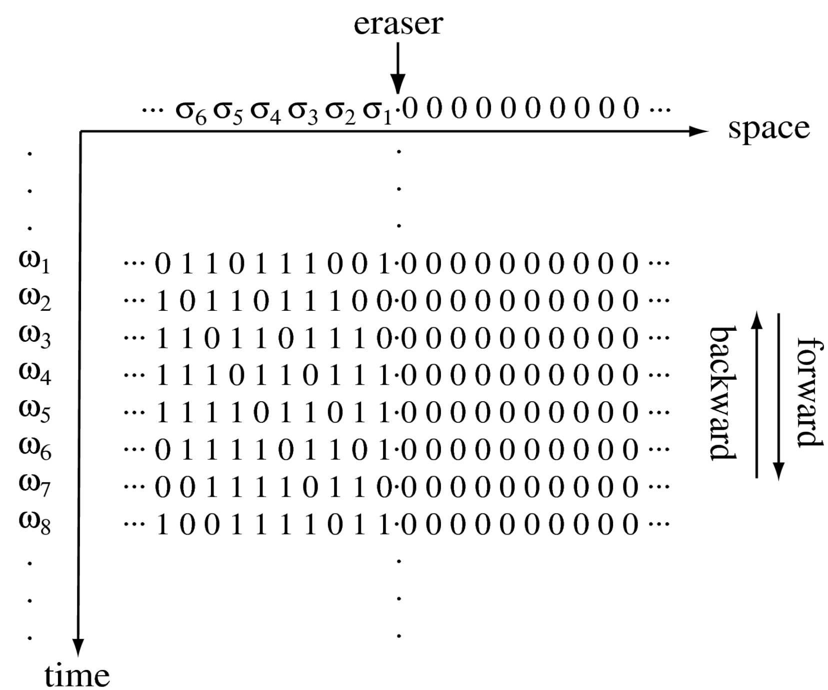
which is also its information content ($I \leq \ln 2$)

What is the minimal cost to erase this sequence?

Erasure process

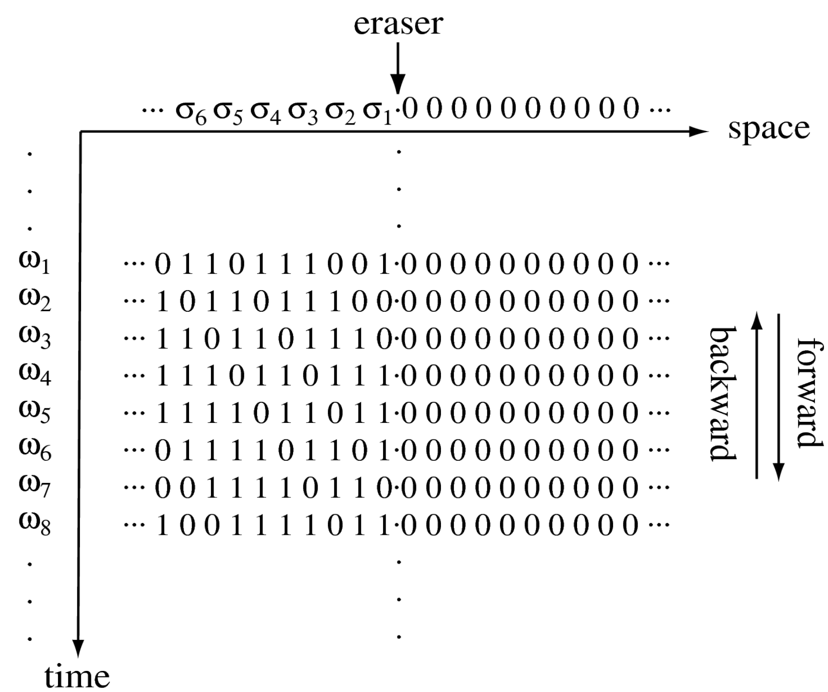


Erasure process



h=0 since it is deterministic. When viewed in reversed time, information is generated at rate I.

Erasure process



h=0 since it is deterministic. When viewed in reversed time, information is generated at rate *I*. Consequently,

$$\Delta_{\mathrm{i}} S = k_{\mathrm{B}}(h^{\mathrm{R}} - h) = k_{\mathrm{B}} I \ \text{per bit}$$

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- Dynamical randomness links information entropy and physical entropy $\longrightarrow$ generalized Landauer's principle

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