Plane Couette flow at the laminar–turbulent transition

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transition to turbulence ??? $\rightsquigarrow$ **closed** *vs.* **open** flows

closed flows (e.g. convection) $\rightsquigarrow$ confinement effects

$\rightsquigarrow$ confined *vs.* extended

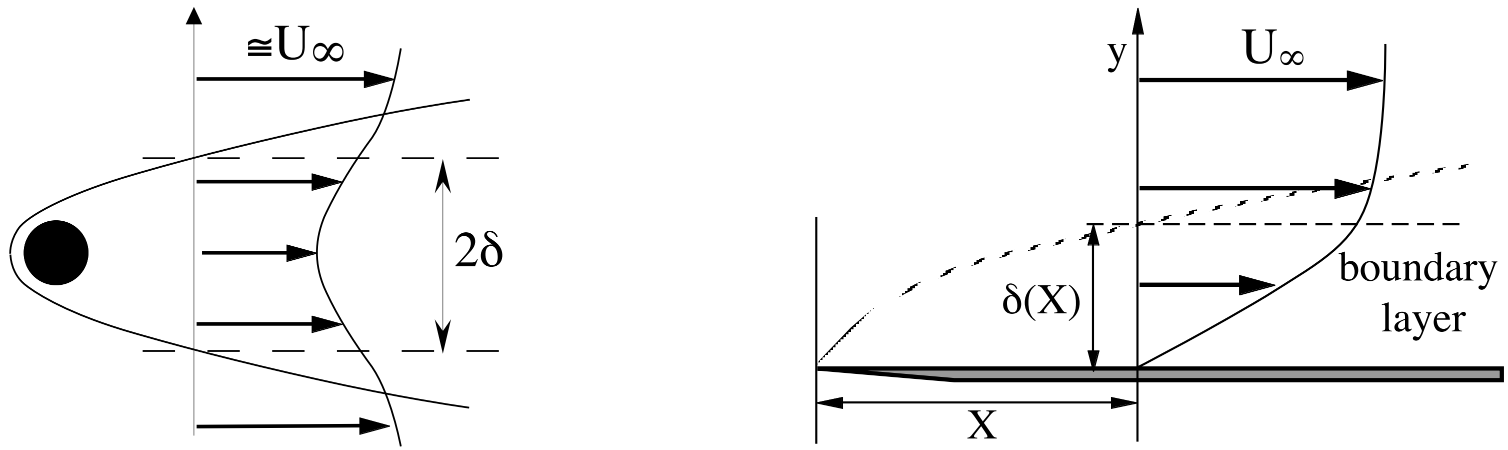
$\rightsquigarrow$ temporal *vs.* spatio-temporal chaos

$\rightsquigarrow$ pretty well understood

open flows less well understood
(even apparently simplest case of parallel flows)

• *linear stage* $\rightsquigarrow$ standard **stability analysis**

$\rightsquigarrow$ inflectional *vs.* non-inflectional base profiles



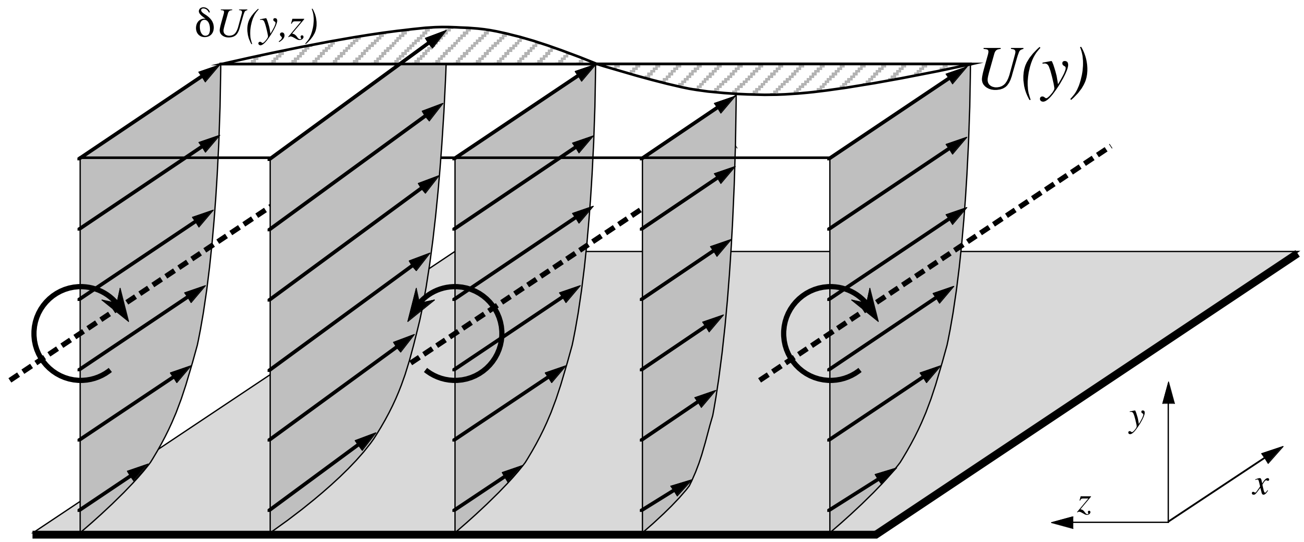
$$\text{Re} = \Delta U \ell/\nu = (\Delta U/\ell) \times (\ell^2/\nu) = \tau_v/\tau_a$$

◇ inflectional $\rightsquigarrow$ linear instability (inertial), e.g. wake
globally super-critical transition to turbulence at low Re

◇ non-inflectional $\rightsquigarrow$ no instability at low Re, e.g. boundary layer
possible viscous instability at large $\text{Re} = \text{Re}_{\text{TS}}$
$\rightsquigarrow$ *conditional stability* is generic (nonlin. instab. at $\text{Re} < \text{Re}_{\text{TS}}$)

• specific role of *advection**

physical consequence $\rightsquigarrow$ interaction *mean flow/fluctuation*



$\rightsquigarrow$ induction of **streaks** by **streamwise vortices**

$\rightsquigarrow$ **lift-up** $\rightsquigarrow$ universal perturbation amplification mechanism

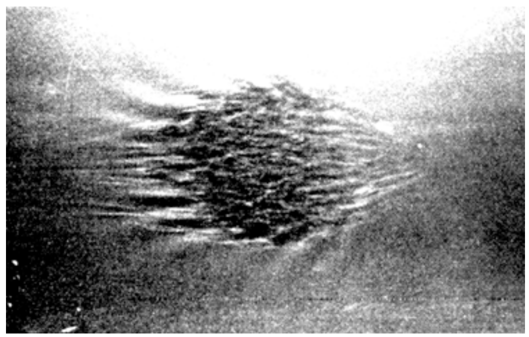
$\rightsquigarrow$ **transient** energy growth even is stable flows (lim. $t \to \infty$)

*see, e.g. P. Schmid, D.S. Henningson,
Stability and Transition in Shear Flows (Springer, 2001)

• direct transition (*by-pass*) to turbulence

induced by transient perturbation growth
in a laminar linearly stable flow $\rightsquigarrow$ nucleation of **turbulent spots**

boundary layer



plane Poiseuille

plane Couette

similar for Poiseuille pipe flow (Ppf)

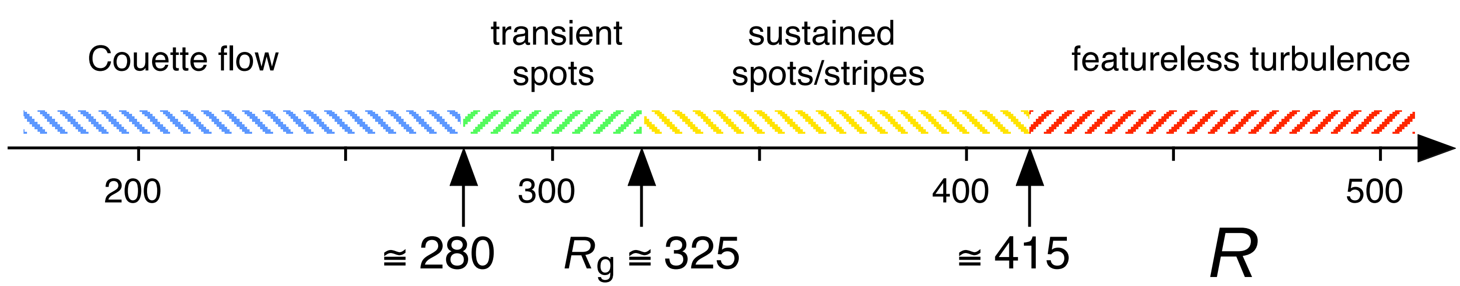
- phenomenology of plane Couette flow (pCf)
  - no linear instability mode
  - no overall advection

experiments: Saclay group (1992–2002)

result:

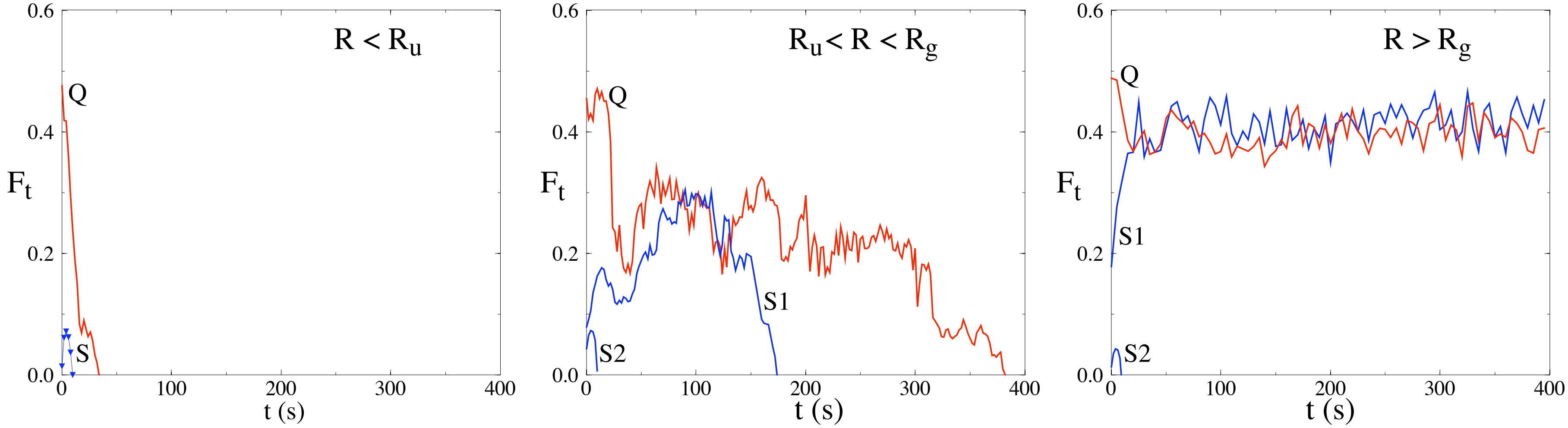
- $\mathrm{Re} < \mathrm{Re_g} \simeq 325 \rightsquigarrow$ global stability of base flow
  $\rightsquigarrow$ systematic return to laminar flow when $t \to \infty$
- $\mathrm{Re} > \mathrm{Re_g} \rightsquigarrow$ regime at $t \to \infty$ *may* be turbulent
  $\rightsquigarrow$ **nonlinear** instability
  against (localized) finite amplitude perturbation

more precisely $\rightsquigarrow$ **bifurcation diagram**



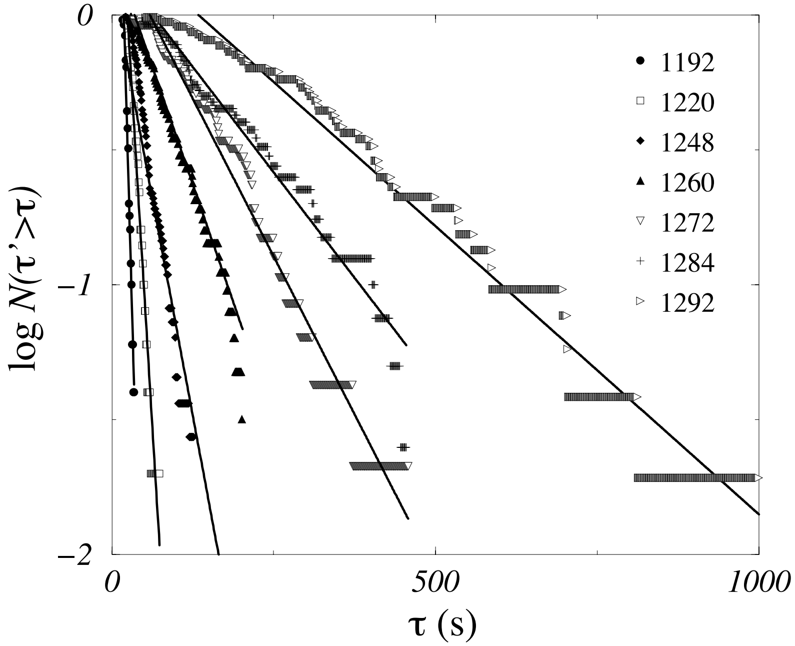
vicinity of $\mathrm{Re_g}$??? $\rightsquigarrow$ experiments (mostly S.Bottin's PhD, 1998):[*]

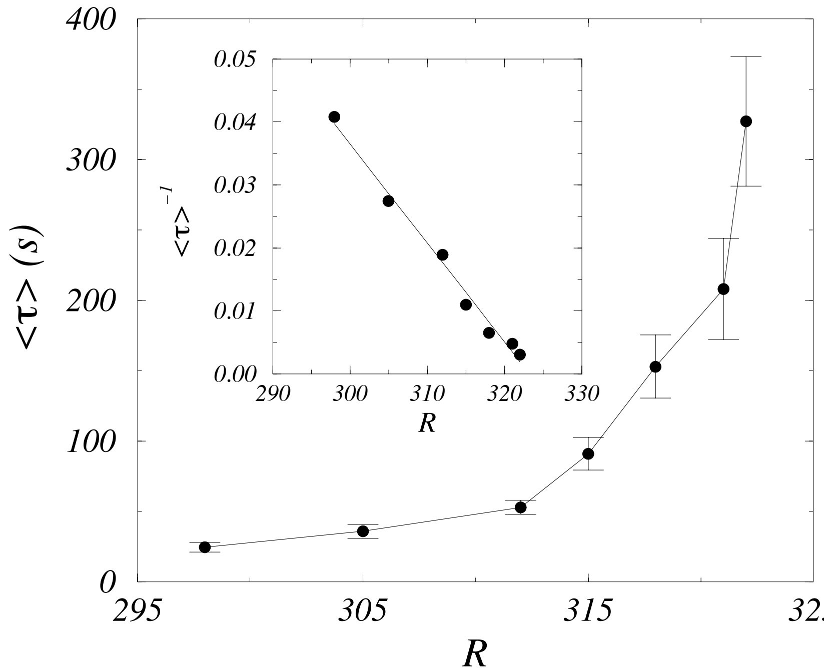
(1) nucleation of spots ("S") $\rightsquigarrow$ amplitude of initial perturbation seems to diverge $A \sim 1/(\mathrm{Re}-\mathrm{Re_g})^\gamma$ as $\mathrm{Re} \to \mathrm{Re_g}_+$ (γ ???) and tend to zero as $A \sim \mathrm{Re}^{-\alpha}$ for $\mathrm{Re} \gg \mathrm{Re_g}$ (α ???)



(2) lifetime of turbulent state prepared at $\mathrm{Re} \gg \mathrm{Re_g}$ quenched ("Q") at $\mathrm{Re} < \mathrm{Re_g}$ $\rightsquigarrow$ transients lifetimes τ $\rightsquigarrow$ distribution $\mathcal{N}(\tau' > \tau) \sim \exp(-\tau/\langle\tau\rangle)$ $\rightsquigarrow$ $\langle\tau\rangle$ increases rapidly as $\mathrm{Re} \to \mathrm{Re_g}_-$

[*]S. Bottin, F. Daviaud, P. Manneville, O. Dauchot, Europhys. Lett. **43** (1998) 171–176.





early suggestion* $\quad \langle\tau\rangle \sim 1/(\mathrm{Re_g} - \mathrm{Re})^{\beta}, \quad \beta \sim 1$

questioned by Hof. *et al.*† who propose $\quad \langle\tau\rangle \sim \exp(b\mathrm{Re}) \quad$ based on (*i*) analogy with results for Ppf and (*ii*) re-analysis of data

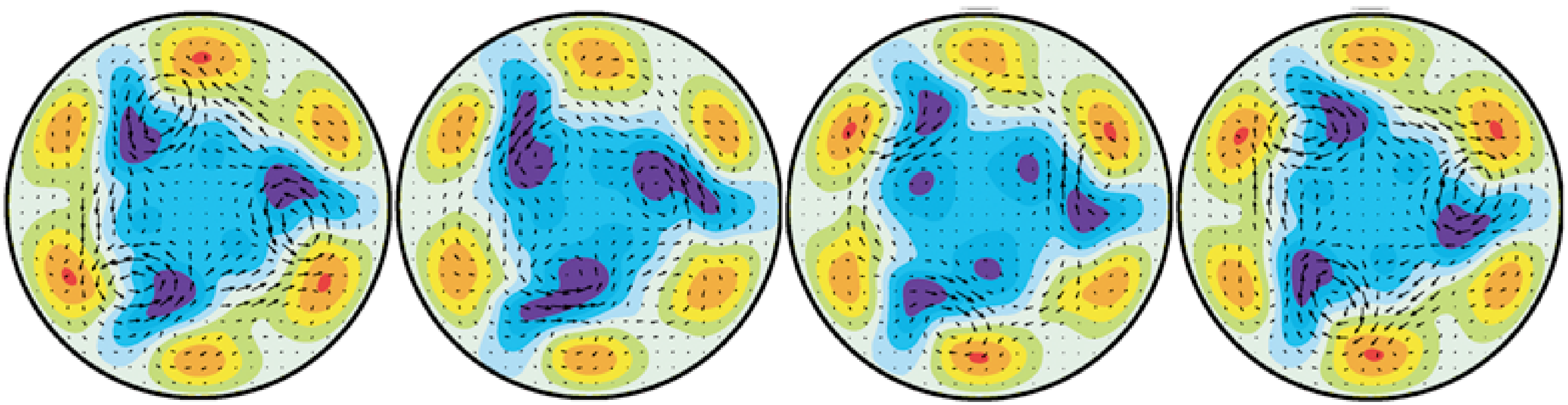
*S. Bottin & H. Chaté, Eur. Phys. J. B **6** (1998) 143–155.

†B. Hof, J. Westerweel, T.M. Schneider, B. Eckhardt, Nature **443** (2006) 59–62.

why ???

Hof *et al.* $\rightsquigarrow$ Ppf transients $\equiv$ chaotic transients associated with **homoclinic tangle** $\rightsquigarrow$ low dim. dynamical systems viewpoint resting on existence of *non-trivial* **unstable** periodic orbits (UPOs)

$\diamond$ such solutions exist in Ppf : Faisst & Eckhardt; Kerswell *et al.**



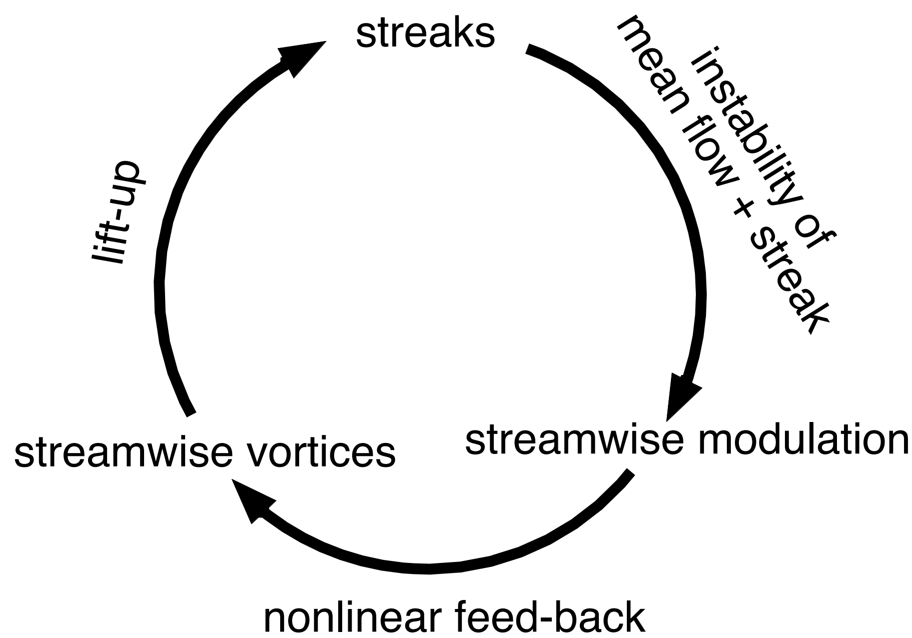
as well as in pCf : Nagata, Clever & Busse†

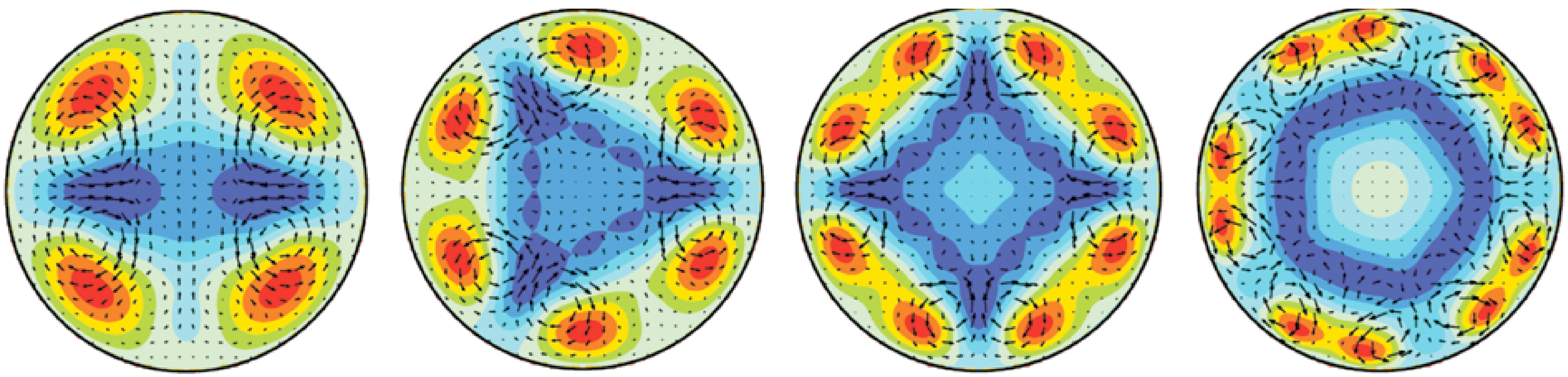
*for a review, see: R.R. Kerswell, Nonlinearity **18** (2005) R17–R44

†M. Nagata, J. Fluid Mech. **217** (1990) 519–527;
R.M. Clever, F.H. Busse, J. Fluid Mech. **344** (1997) 137–153.

not a surprise $\rightsquigarrow$ mechanism ?
cf. "regeneration" cycle:
lift-up + instability
propagation $\rightsquigarrow$ by-product
of instability

analogous situation for pCf

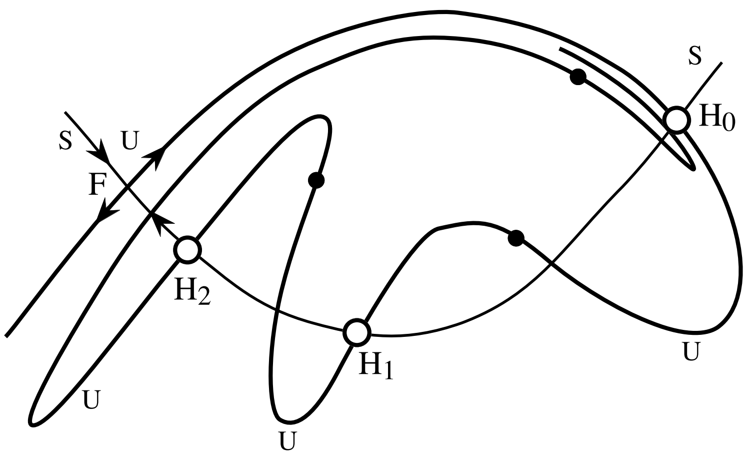


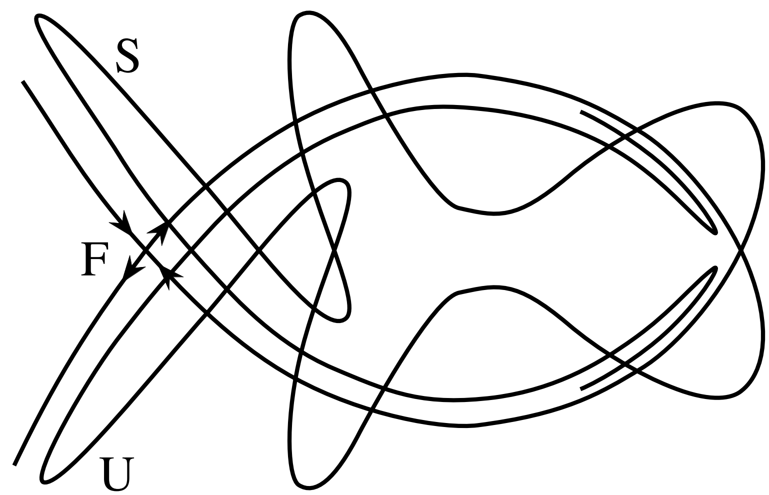


fugitively observed in experiments
see: Hof *et al.*, Science **305** (2004) 1594–1598.

• **homoclinic tangle** ???

unstable periodic orbit with stable and unstable manifolds
1 transverse intersection ⇒ uncountable infinity of intersections (Poincaré)





chaotic repellor (invariant set of homoclinic points)
⇝ chaotic transients around it
⇝ exponential distribution of transient lifetimes
⇝ variation of decrement with control parameter ???

• Ppf $\rightsquigarrow$ case not completely settled*

exponential transient length distribution with decrement $\searrow 0$

– either as (Re$_g$ – Re) for Re $\to$ Re$_{g-}$ ($\sim$ critical behavior)

– or as exp($-b$Re) as Re $\nearrow$

possible origin of discrepancies:

– role of experimental conditions ($\Delta P/\Delta x$ =cst. or cst. flux)

– finite time/size effects

$\rightsquigarrow$ is the analysis in terms of low dim. dynam. syst. relevant?

temporal chaos OK if system is **0D** but Ppf is **quasi-1D**

• pCf $\rightsquigarrow$ beyond phenomenology ???

modeling in a deliberately spatiotemporal perspective

to accounting for **quasi-2D** feature

$\rightsquigarrow$ personal work in coll. with M. Lagha (PhD thesis, 2006)

*Peixinho & Mullin, Phys. Rev. **96** (2006) 094501; Willis & Kerswell, Phys. Rev. Lett. **98** (2007) 014501; Hof *et al.*, Nature **443** (2006) 59–62.

• modeling $\rightsquigarrow$ low dimensional $\Rightarrow$ freeze all the space dependence
$\rightsquigarrow$ ODEs governing a small set of amplitudes, cf. Lorenz model
similar spirit for open flows $\rightsquigarrow$ Waleffe models
$\rightsquigarrow$ well adapted only to **confined** systems
(or systems with periodic b.c. at "short" distances)

$\rightsquigarrow$ freeze cross-stream dependence, let in-plane dependence free
$\rightsquigarrow$ partial differential equations, cf. Swift–Hohenberg model
$\rightsquigarrow$ adapted to **extended** systems

use Galerkin method to obtain model (2.5D)
from primitive (3D) equations

previous work $\rightsquigarrow$ stress-free b.c. at the plates
$\rightsquigarrow$ interesting but unrealistic $\rightsquigarrow$ realistic no-slip b.c.

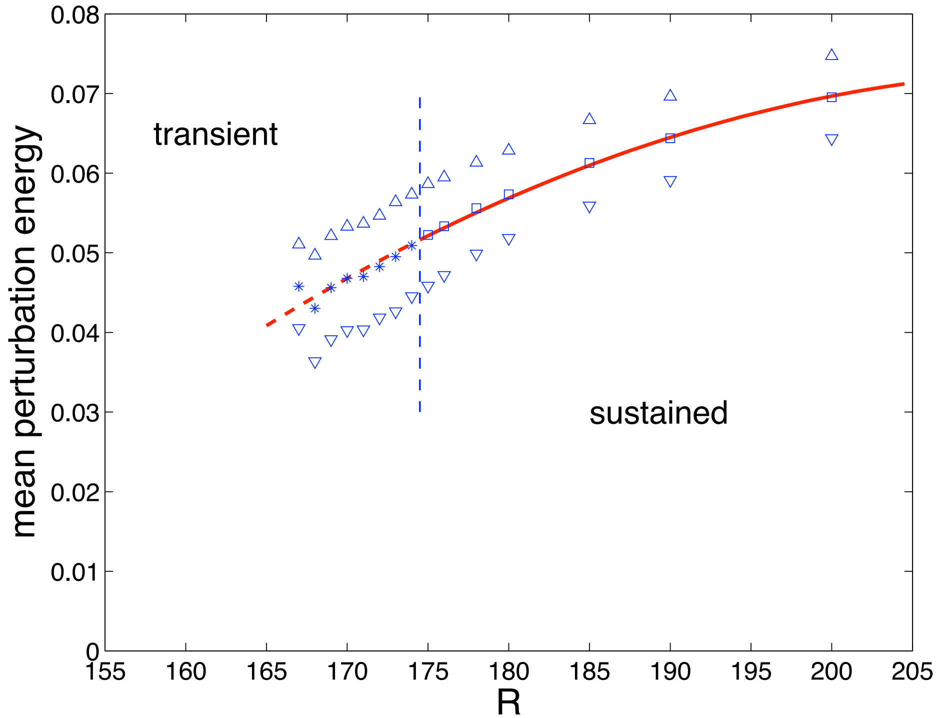
explicit expression $\rightsquigarrow$ last slides (if someone is interested)

- *a priori* relevant general features
  - non-normal linear terms including lift-up mechanism
  - linear viscous damping
  - nonlinear advection terms preserving perturbation kinetic energy
  - linearly stable base flow for all Re
- *a posteriori* relevant features (from numerical simulations)
  - extensivity of homogeneous turbulent state
  - sub-critical "laminar $\leftrightarrow$ turbulent" transition ($\mathrm{Re_g}$???)
  - transient states with exponentially decaying lifetime distribution
  - turbulent spots resemble what is experimentally observed

$\rightsquigarrow$ present results relevant to the "critical/exponential" controversy
$\rightsquigarrow$ define dimensionless system's size
$\rightsquigarrow$ aspect ratio $\Gamma_x = L_x/d$, $\Gamma_z = L_z/d$, $d \equiv$ gap,
$\Gamma = \Gamma_x \times \Gamma_z$, here numerical experiments (periodic b.c.)
  - at moderate aspect-ratio $\Gamma = 16 \times 16$ ($\mathcal{D} = 32 \times 32 \times 2$)
  - at large aspect ratio $\Gamma = 128 \times 64$ ($\mathcal{D} = 256 \times 128 \times 2$)

compare to laboratory experiments $\rightsquigarrow$ typically 190×35
and to observed internal scale: coherent streak segments $\sim 6 \times 3$

$\diamond$ sub-criticality ($\Gamma = 16 \times 16$, adiabatic decrease of R)



transition <u>transient $\longrightarrow$ "sustained"</u> at Re $\simeq 175 \simeq$ Re$_g$

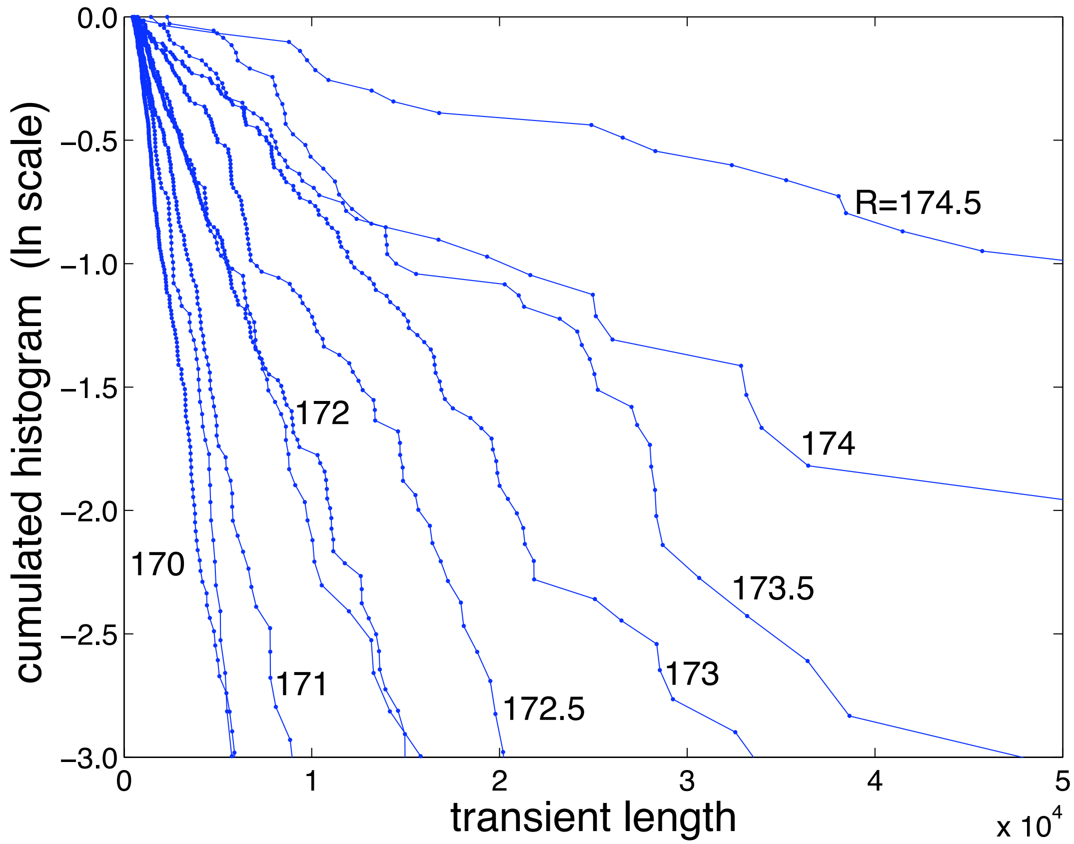
Re$_g^{\text{model}} \sim 0.5Re_g^{\text{lab.}}$ $\rightsquigarrow$ viscous dissipation and energy transfer to <u>cross-stream small scales</u> underestimated (truncation)

but qualitative spatio-temporal features are preserved

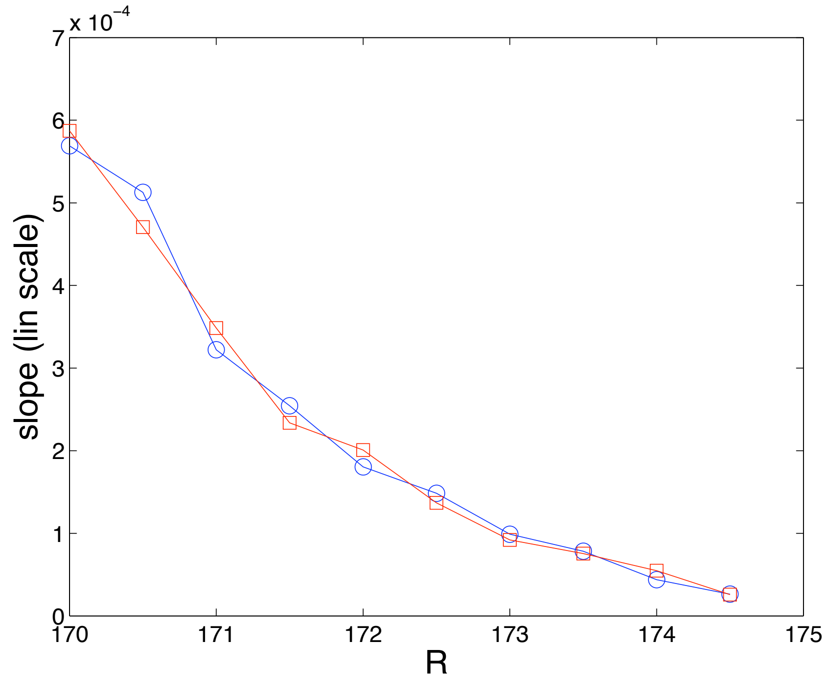
study first the **decay** transition <u>turbulent $\longrightarrow$ laminar</u>

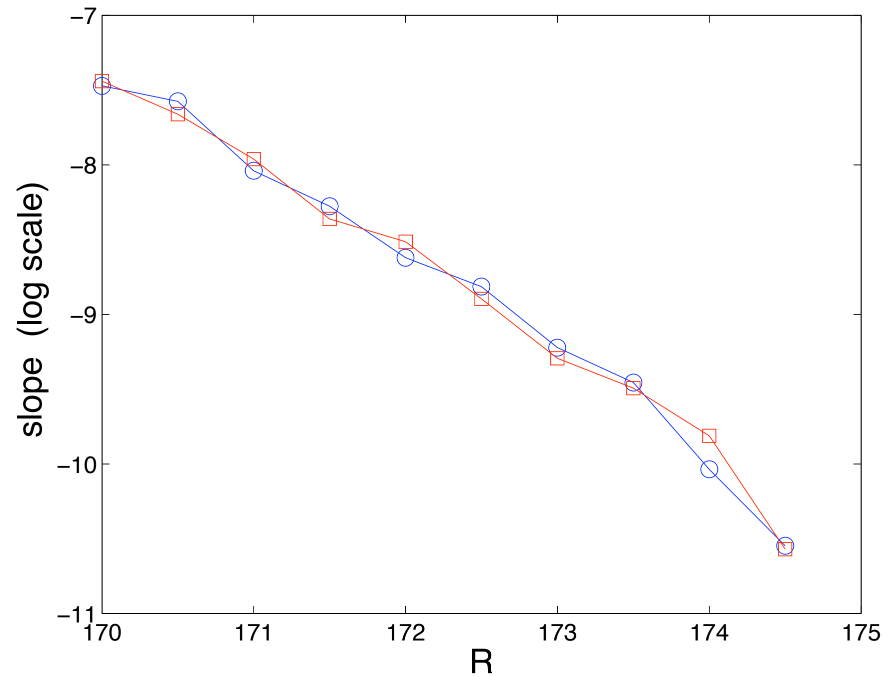
◇ transients ($\Gamma = 16 \times 16$)

Q-type experiments: state prepared at $\mathrm{Re_i} = 200$
Re decreased to $\mathrm{Re_f} < \mathrm{Re_g} \ll \mathrm{Re_i}$



$\leadsto$ variation of slopes with Re ???





– exponential decrease of slopes, hence $\langle\tau\rangle \sim \exp(b\mathrm{Re})$)

– off-aligned points at Re $= 174$ and 174.5 suggest <u>cross-over</u> to critical behavior very close to Re $= 175$

statistical improvement beyond reach of numerical means used for that experiment $\rightsquigarrow$ explanation ???

visualizations for $\Gamma = 16 \times 16$ do not discriminate <u>temporal</u> from <u>spatio-temporal</u> behavior temporal is likely in view of size of streak segments compared to Γ

$\rightsquigarrow$ consider a larger domain $\rightsquigarrow$ $\Gamma = 128 \times 64$ (8×4 times larger)

result: turbulent state can be maintained over large time periods well below $R = R_{\mathrm{g}} = 175$ $\rightsquigarrow$ expensive to study numerically
$\rightsquigarrow$ limited number of trials $\rightsquigarrow$ no direct statistics
(experimentalists do a better job, but with other limitations)

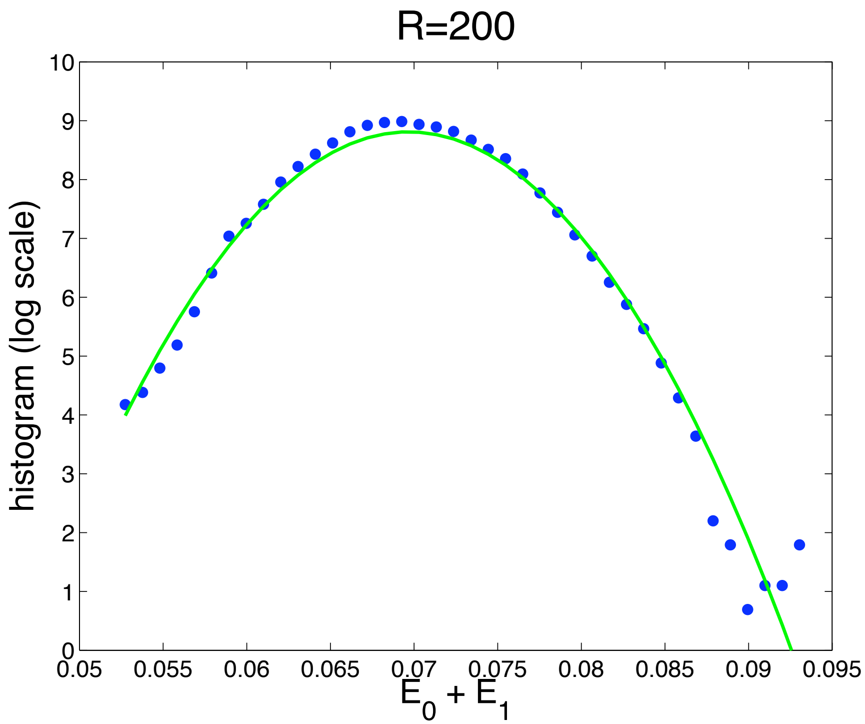
- **video of quench at $\mathbf{R_f = 167}$**

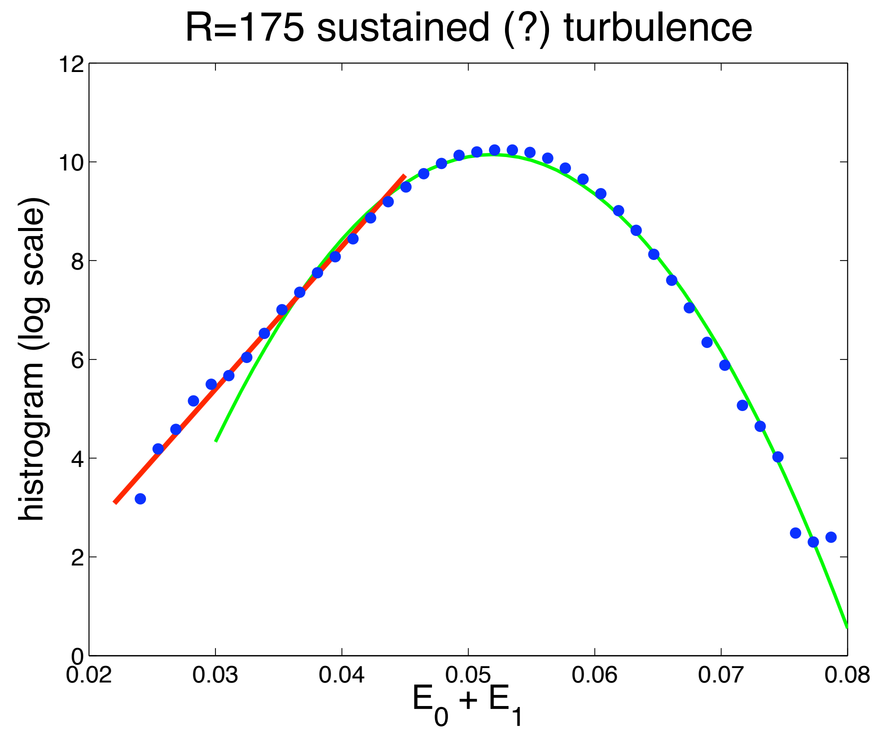
$\rightsquigarrow$ nucleation of laminar domains that expand
$\rightsquigarrow$ late stage is a retraction of the turbulent domain

$\rightsquigarrow$ suggests that, for $\Gamma = 16 \times 16$, last stage is also a retraction
$\rightsquigarrow$ turn the question to "when does the transient begins ?"
$\rightsquigarrow$ Pomeau's idea of **nucleation** expected from the connection
between a globally **sub-critical** bifurcation
and a **first-order** thermodynamic phase transition*

*in: Bergé, Pomeau, Vidal, *L'Espace Chaotique* (Hermann, 1998) Chapter IV.

test the nucleation idea ? $\rightsquigarrow$ return to $\Gamma = 16 \times 16$

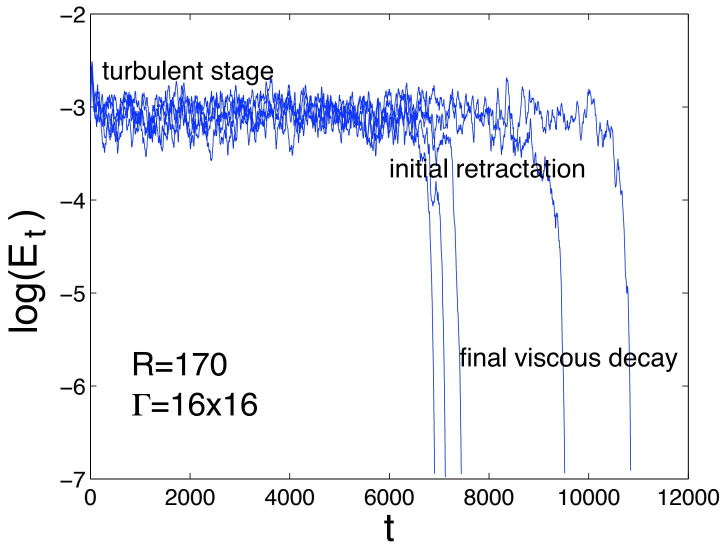


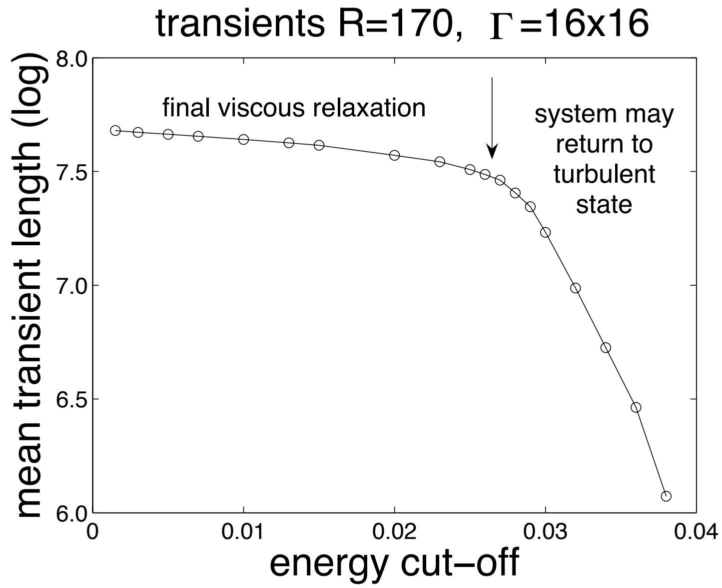


– Re $=$ 200 $\rightsquigarrow$ Gaussian histogram $=$ incoherent superposition
chaotic mixture of laminar and chaotic small structures

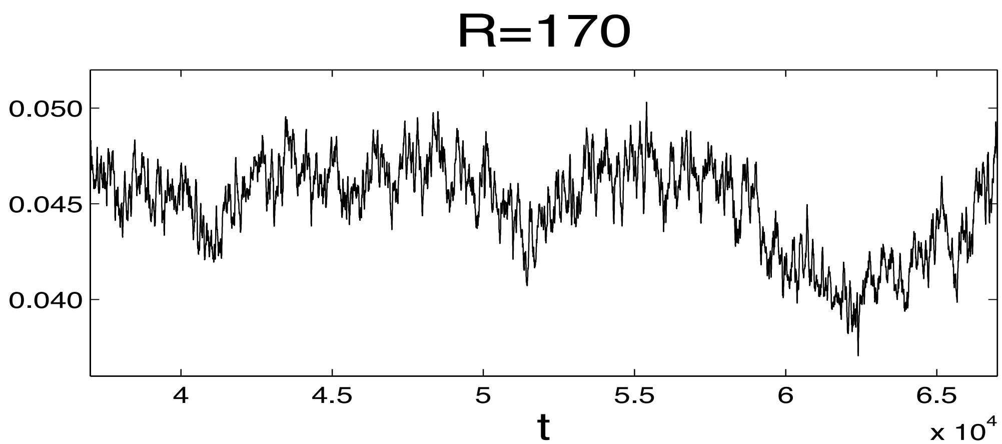
– Re $=$ 175 $\simeq$ Re$_\mathrm{g}$ $\rightsquigarrow$ max shifts $\searrow$; exponential tail at low energy
coherent **large deviation** $\rightsquigarrow$ **germ** that grows if large enough
$\rightsquigarrow$ irreversible decay to laminar stage when $E_\mathrm{t} < E_\mathrm{lim}$

evidence for $E_{\lim} \simeq 0.025$ ($\ln E_{\lim} \simeq -3.7$) at moderate Γ





back to the wide system ⇝ long time series



• **video 1** $\rightsquigarrow \mathbf{R = 170}$, **full resolution** $\mathbf{59000 < t < 67000}$

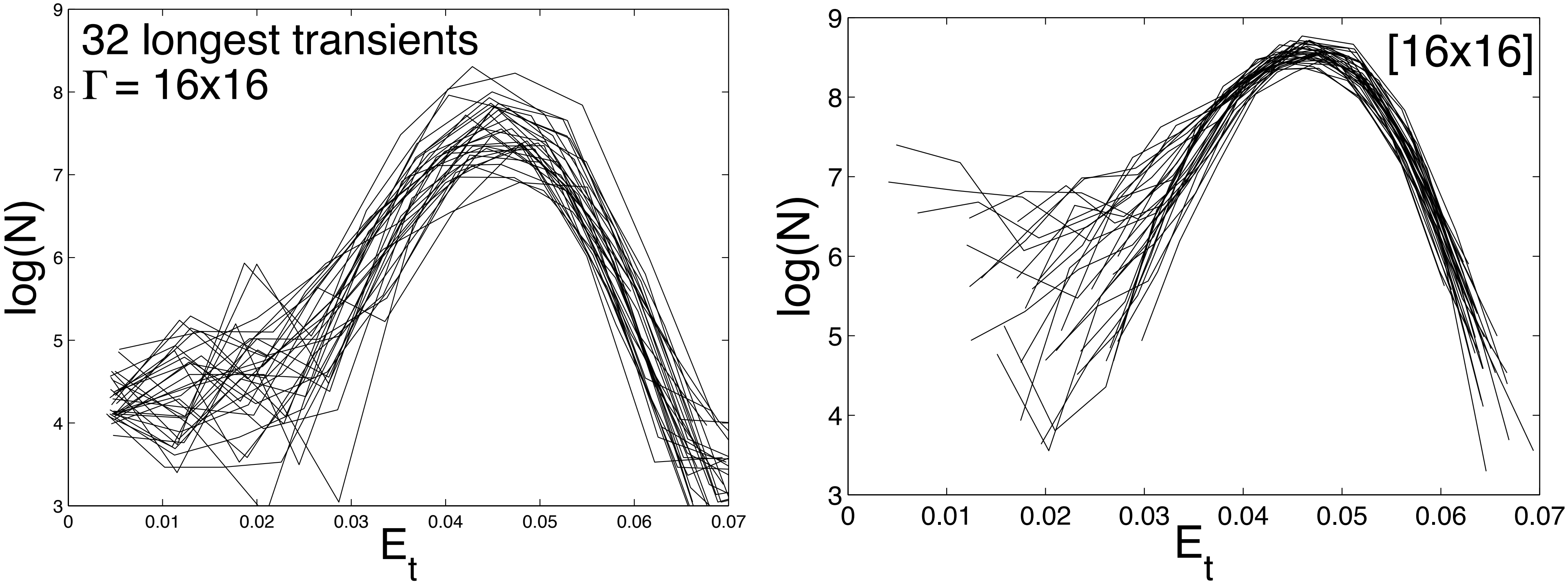
shows existence of large laminar domains that last very long
$\rightsquigarrow$ wait to see the system decay ???

$\rightsquigarrow$ compare to small system ($\Gamma = 16 \times 16$)

• **video 2** $\rightsquigarrow \mathbf{R = 170}$, **"low" resolution** $\mathbf{47000 < t < 67000}$

obtained by binning original large domain into squares 8×8
further grouped to give larger rectangular or square sub-domains,
i.e. 16×16 to be used for comparison with $\Gamma = 16 \times 16$ system

yields individual time series analogous to those of smaller systems
$\rightsquigarrow$ construct histograms



$\rightsquigarrow$ the 16×16 system "dies" at the end of a transient ($E_t < E_{\text{lim}}$) while a given $[16 \times 16]$ sub-domain of the 128×64 system that become laminar can "resuscitate" by contamination from turbulent neighbors

$\rightsquigarrow$ first guess : convert frequency of laminar domains of given size in the 128×64 system into transient length distribution for the 16×16 system ($\rightsquigarrow$ may need correction due to subtle size effects) size effects ??? $\leadsto$ long-range processes linked to pressure (present in the model $\neq$ more simplified models not directly derived from NS equations, e.g. CMLs)

best seen when studying hysteresis at the transition

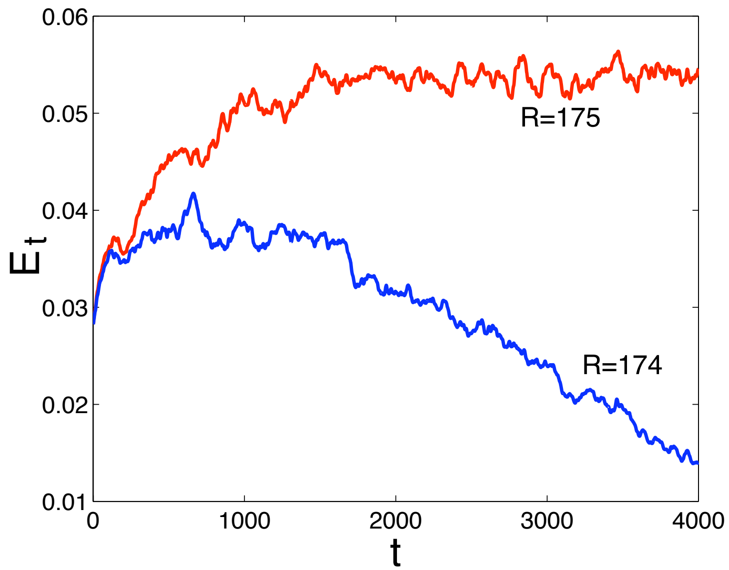
up to now <u>turbulent $\rightarrow$ laminar</u> transition *via* nucleation and development of laminar patches

now <u>laminar $\rightarrow$ turbulent</u> $\leadsto$ starting point ?

a) localized spot $\leadsto$ two parameters: extension and intensity
$\leadsto$ systematic study left for future work

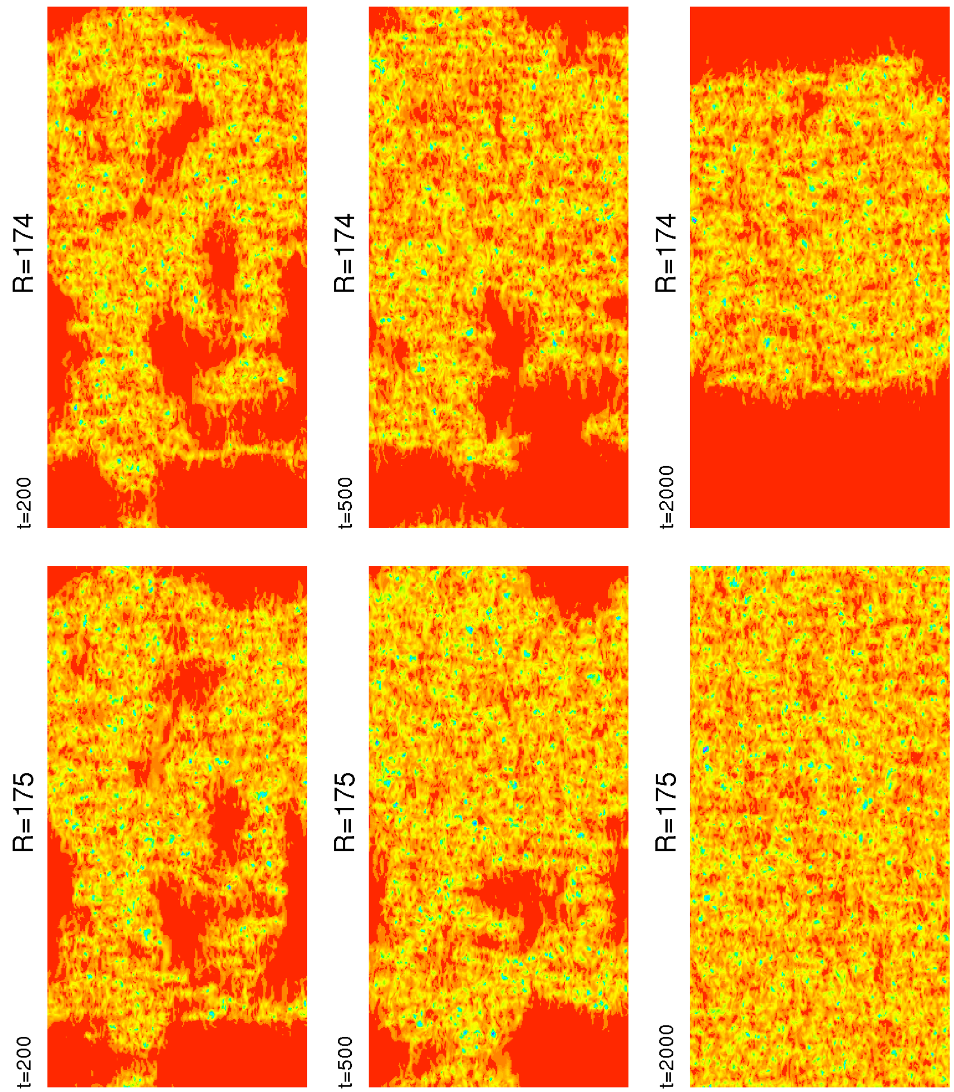
b) more or less homogeneous low amplitude "noise"
$\leadsto$ to be presented (briefly)

relevant “noise” obtained by “attenuating” a turbulent solution

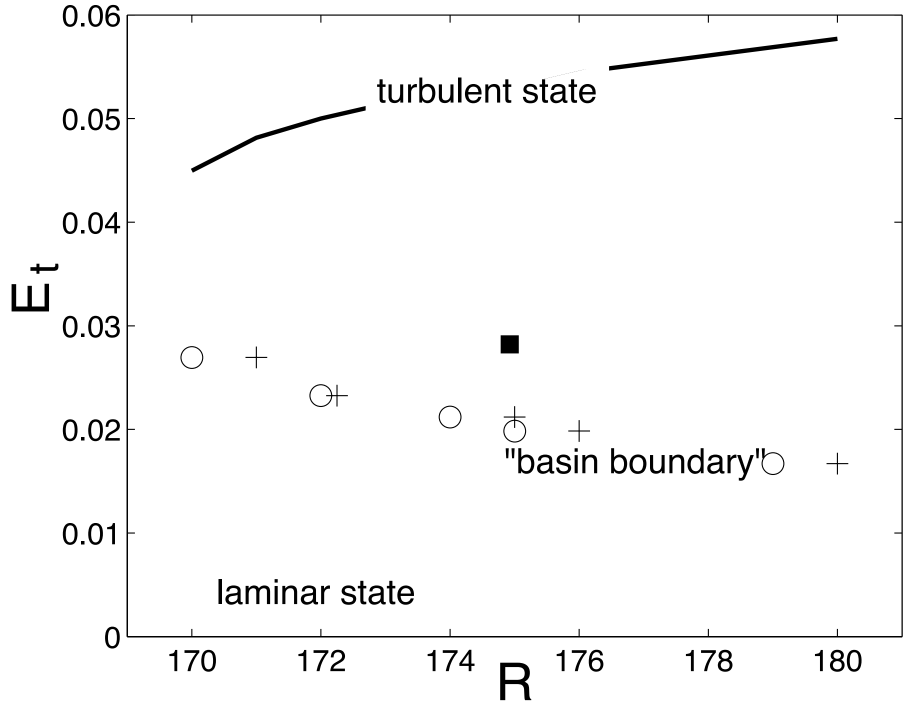


refine to determine
“edge of turbulence” $\rightsquigarrow$
with this i.c.
$174.925 < R < 174.95$

change i.c.?



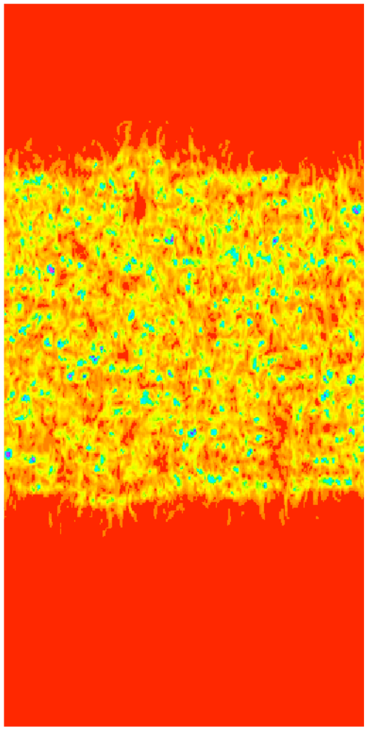
change energy contents of i.c. at given R or change R at given i.c.
"noisy i.c." $\rightsquigarrow$ rough bifurcation diagram
but "basin boundary" depends on i.c. amplitude and homogeneity

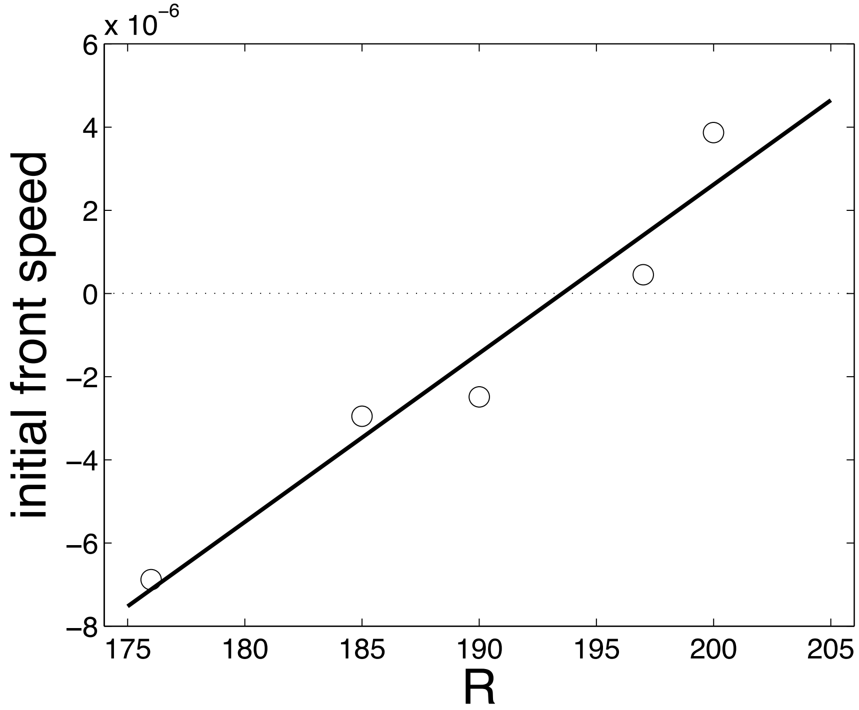


very low energy noisy i.c. ???

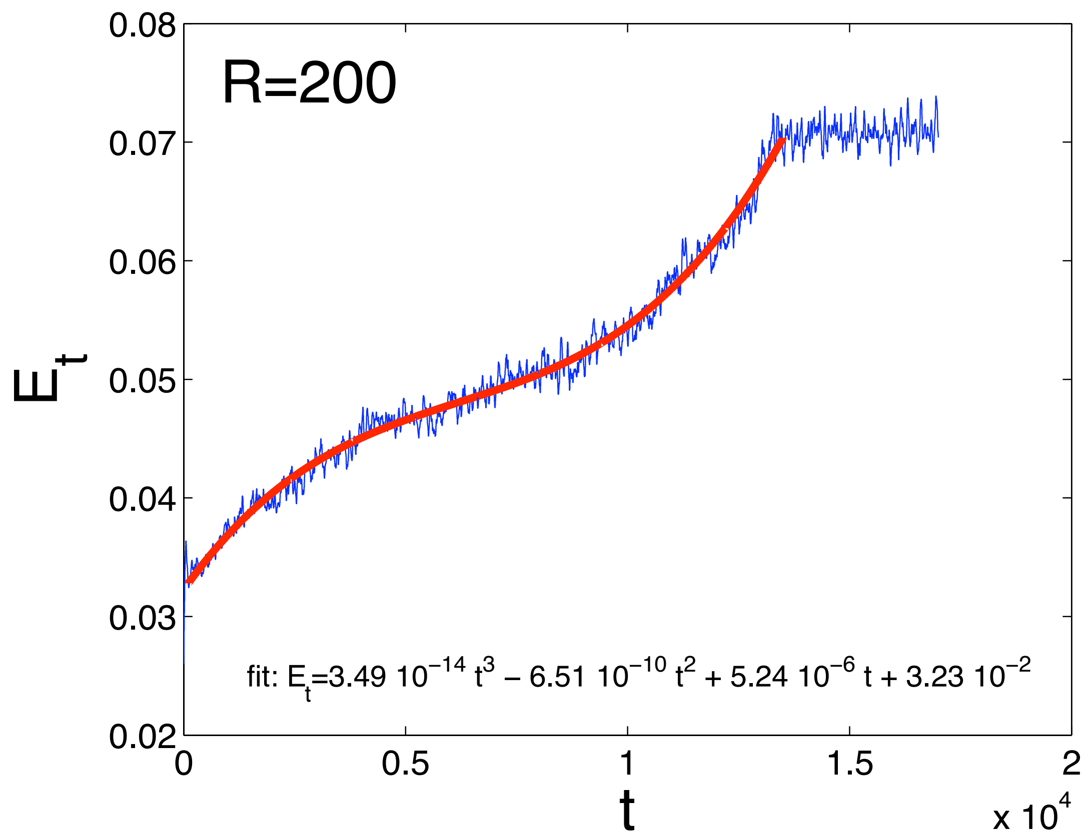
$\rightsquigarrow$ different transition due to different early stage:
initial smoothing
$\rightsquigarrow$ leaves few germs
$\rightsquigarrow$ germs grow if R large
$\rightsquigarrow$ next form transverse bands
$\rightsquigarrow$ final turbulent invasion stage
if R significantly above 200

$\rightsquigarrow$ study laminar–turbulent coexistence and fronts
produce a banded i.c. and change R





evidence of non-local effects $\rightsquigarrow$ speed depends on turbulent fraction



interpretation is delicate : periodic b.c. influence band orientation but role of instantaneous turbulent/laminar global pattern is obvious both for onset and decay of turbulence

conclusion

- context: sub-critical transition to turbulence
  $\rightsquigarrow$ more specifically Ppf & pCf $\rightsquigarrow$ similar (same ?) problem
- origin of difficulties: nature of the non-trivial solution
  competing with the base state
- answers ? $\rightsquigarrow$ dynamical systems and chaos
  stems from temporal analysis valid for confined system
  $\rightsquigarrow$ classical theory of chaotic transients
  existence of unstable periodic solutions + tangle
  these solutions exist (calculated/observed) but is this enough ?
    - pipe Poiseuille flow $\rightsquigarrow$ quasi 1D
    - plane Couette flow $\rightsquigarrow$ quasi-2D

  $\rightsquigarrow$ Pomeau (1986, 1998) $\rightsquigarrow$ nucleation problem in connection
  with first-order (thermodynamic) phase transitions

• modeling approach $\rightsquigarrow$ dimensional reduction **in physical space**
$\rightsquigarrow$ different from standard dynamical-system viewpoint

low-order truncation of a Galerkin projection of NS equations
– negative feature : energy transfer through cross-stream (small) scales underestimated $\rightsquigarrow$ lowered transitional range

– positive aspect $\rightsquigarrow$ correctly extract energy from base flow through interplay of streamwise vortices and streaks (large in-plane structures) $\rightsquigarrow$ qualitatively reproduces hydrodynamical features (e.g. non-local pressure effects) and transition properties

• even in absence of firm conclusions, most interesting results :

$\diamond$ better appreciation of drawbacks and virtues of dynamical system approach and phase transition viewpoint :
$\rightsquigarrow$ reinterpretation of transient length distribution

$\diamond$ glimpse on origin of complications : size effects and role of topology of laminar/turbulent domains

$\diamond$ suggests to look at Ppf along same lines (quasi-1D $\neq$ 0D)

- two levels of open questions and perspectives
  - ◇ immediate, concrete, hydrodynamical consequences for other **globally sub-critical** flows experiencing wild transition to turbulence *via* streaks, streamwise vortices, spots. . . and for **transition control**
  - ◇ abstract and general: role of noise and statistics $\rightsquigarrow$ **nature** of the turbulent attractor and **thermodynamic** approach to far-from-equilibrium systems theory in continuous media

Acknowledgments

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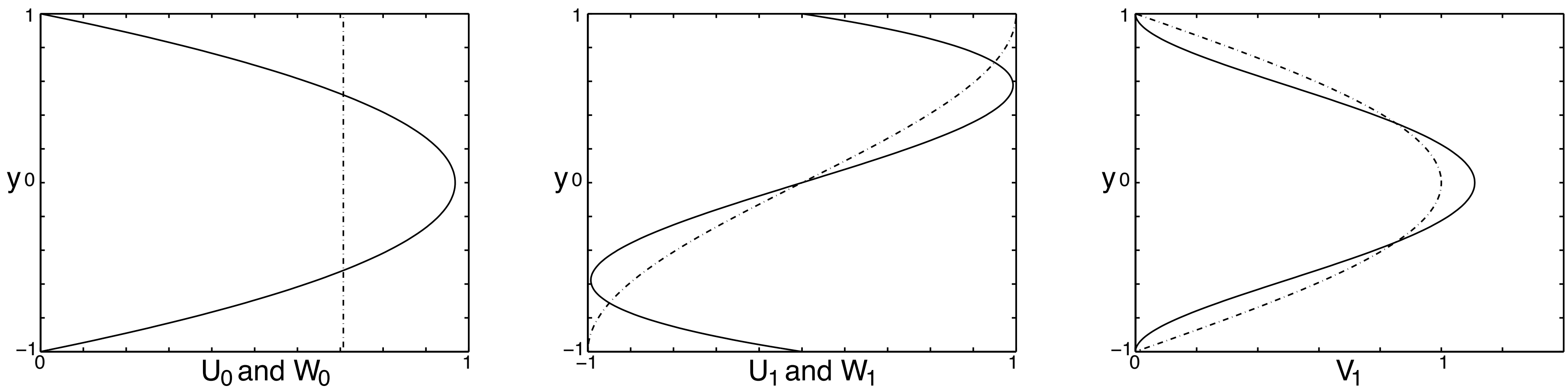
IDRIS (Orsay) projects #61462, 72138

- **the model**

$\rightsquigarrow$ base flow $\quad u = u_{\mathrm{b}}(y) = y$

polynomial expansion of perturbations (here lowest order trunc.)

$$\left\{u', w'\right\} = \{U_0(x,z,t), W_0(x,z,t)\}\, B(1-y^2) + \{U_1, W_1\}\, Cy(1-y^2)$$
$$v' = V_1(x,z,t) A(1-y^2)^2$$



anticipated to be good enough since

– perturbations known to occupy the full gap for Re $\sim$ Re$_{\mathrm{g}}$

– no-slip functions dissipate more than stress-free basis functions

– Galerkin expansion possible (but tedious) at higher orders

- continuity equation

$$\partial_x u' + \partial_y v' + \partial_z w' = 0$$

by projection $\rightsquigarrow$

◇ <u>even part</u> (*streaks* $\rightsquigarrow \{U_0(z)\}$)

$$\partial_x U_0 + \partial_z W_0 = 0$$

◇ <u>odd part</u> (*streamwise vortices* $\rightsquigarrow \{V_1(z), W_1(z)\}$)

$$\partial_x U_1 - \beta V_1 + \partial_z W_1 = 0 \qquad\qquad \beta = \sqrt{3} \approx 1.73$$

- linear momentum

$$\partial_t \mathbf{v}' + \mathbf{v}' \cdot \boldsymbol{\nabla} \mathbf{v}' = -\boldsymbol{\nabla} p' - u_\mathrm{b} \partial_x \mathbf{v}' - v' \frac{\mathrm{d}}{\mathrm{d}y} u_\mathrm{b} \hat{\mathbf{x}} + \nu \boldsymbol{\nabla}^2 \mathbf{v}'$$

◇ <u>in-plane, even part</u> (streamwise only, spanwise similar)

$$\partial_t U_0 + N_{U_0} = -\partial_x P_0 - a_1 \partial_x U_1 - a_2 V_1 + \mathrm{Re}^{-1} (\partial_{xx} + \partial_{zz} - \gamma_0) U_0$$

$$N_{U_0} = \alpha_1 (U_0 \partial_x U_0 + W_0 \partial_z U_0) + \alpha_2 (U_1 \partial_x U_1 + W_1 \partial_z U_1) + \alpha_3 V_1 U_1$$

◇ <u>in-plane, odd part</u> (streamwise only, spanwise similar)

$$\partial_t U_1 + N_{U_1} = -\partial_x P_1 - a_1 \partial_x U_0 + \mathrm{Re}^{-1} (\partial_{xx} + \partial_{zz} - \gamma_{1||}) U_1$$

$$N_{U_1} = \alpha_2 (U_0 \partial_x U_1 + U_1 \partial_x U_0 + W_0 \partial_z U_1 + W_1 \partial_z U_0) - \alpha_4 V_1 U_0$$

◇ <u>wall-normal</u>

$$\begin{aligned} \partial_t V_1 + N_{V_1} &= -\beta P_1 + \mathrm{Re}^{-1} (\partial_{xx} + \partial_{zz} - \gamma_{1\perp}) V_1 \\ N_{V_1} &= \alpha_5 (U_0 \partial_x V_1 + W_0 \partial_z V_1) \end{aligned}$$

all coefficients combinations of integrals in the form

$$J_{n,m} = \int_0^1 y^n (1 - y^2)^m \mathrm{d}y = \sum_{k=0}^m \binom{k}{m} \frac{(-1)^k}{2k+n+1}$$